The U.S. Westward Expansion *

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Abstract

The U.S. economic development in the nineteenth century was characterized by the westward movement of population and the accumulation of productive land in the West. This paper presents a model of migration and land improvement to identify the quantitatively important forces driving these phenomena. The conclusion is that the decrease in transportation costs induced the westward migration, while population growth was responsible for the investment in productive land.

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1 Introduction

The United States of 1900 differed dramatically from the country created after the Revolutionary War. The first prominent difference was size. In 1800 the U.S. was less than one million square miles, while in 1900 it encompassed about three million square miles. A consequence of this territorial expansion was the significant growth in the stock of productive land. Between 1800 and 1900 the stock of land was multiplied by 14, that is an annual growth rate of 2.7 percent – see Figure 1. The second difference was the change in the geographic distribution of population. In 1800, less than seven percent lived in the West. By 1900 this number increased to roughly 60 percent – see Figure 2. The combination of these two facts constitutes the Westward Expansion. The geographical shift of economic activity also captures the Westward Expansion. In 1840, the West accounted for less than 30 percent of total personal income whereas in 1900 this share rose to 54 percent (and remained stable at about 60 percent ever since).

The Westward Expansion was a part of the growth experience of the United States. From this perspective, the present paper contributes to the literature addressing phenomena such as the demographic transition and the structural transformation.[1] It is also interesting to note that the Westward Expansion did not affect only the United States. During the nineteenth century, 60 million European migrated to the new world. Most were attracted by the economic opportunities they expected to find there and, in particular, the possibilities to acquire land in the western part of the United States. In fact, the Westward Expansion is a phenomenon similar to the international immigration to the United States as a whole.

This paper proposes an investigation of the quantitatively important forces driving the Westward Expansion. The focus is on the time path of the geographic distribution of population and the accumulation of productive land. Specifically, the question is: What forces can account for the magnitude and pace of the westward movement of population and accumulation of land, during the nineteenth century?

The strategy is the following. Section 2 presents the facts about population movement and productive land, and discusses the forces that might have caused the Westward Expansion. The forces under consideration are population growth and technological progress in various activities, such as transportation, production and the development of productive land. Section 3 presents a model incorporating these forces and, for clarity, proceeds in two steps. First, Section 3.1 develops and analyzes a static model in which the mechanisms are as transparent as possible. However, for the quantitative question at hand, a dynamic model is a more appropriate tool for three reasons. First, migration was a major feature of the Westward Expansion and one can easily differentiate it from natural increase in a dynamic setting. Second, investment in land was another major aspect which needs to be modeled dynamically. Finally, a dynamic model allows one to compute transition paths, and thus analyze the *pace* of the

[1] Contributors to this literature are, for instance, Greenwood and Seshadri (2002) and Caselli and Coleman (2001).

Westward Expansion. Section 3.2 presents such a model. Section 4 presents a computational experiment consisting of two steps. In the first, the procedure used to assign numerical values to the parameters of the dynamic model is described. Part of this procedure implies that the model is matched to the data from the onset. The findings of the quantitative analysis of the model are, therefore, described in the second step, through a set of counterfactual experiments. Each experiment consists in shutting down one driving force at a time, recomputing the transition path and assessing the departure from the baseline results. The conclusion of this exercise is that the decrease in transportation costs induced the westward migration, while population growth was responsible for the investment in productive land. Section 5 concludes.

2 Facts and Hypothesis

2.1 Population

In 1803, at the time when president Thomas Jefferson purchased the Louisiana territory, a small U.S. army unit, lead by Meriwether Lewis and William Clark, headed west across the continent. The goal of the expedition was to find a route to the Pacific ocean using the Missouri and Columbia river systems. Lewis and Clark returned more than 2 years later with their findings concerning the land, its natural resources and its native inhabitants. Their work became most valuable for migrants that, throughout the rest of the century, settled the continent.

The demographic aspect of the Westward Expansion is represented by the increasing share of western population, displayed in Figure 2. It is important to keep in mind that there are two causes leading to an increase in this share: migration and, potentially, an excess rate of natural increase of the western population over the eastern population.

It is difficult to build a consistent measure of internal migration for the nineteenth century because the Census does not report population by state of birth and state of residence before 1850. There is little debate, however, about the existence of such migration. Gallaway and Vedder (1975), for instance, estimate the components of population growth for the "Old Northwest," for the period 1800-1860.[2] They show that between 1800 and 1810, 80 percent of population growth in this region was accounted for by net migration. This percentage was 77 in the 1810s and 50 in the 1820s. Along the same line, Oberly (1986) reports that a third of the veterans of the war of 1812 lived as old men in a more western state than the one where they volunteered to serve.

During the nineteenth century, there was a fertility differential in favor of western regions – see Yasuba (1962). It is not easy to conclude, from this evidence, that the rate of natural increase was higher in the West than in the East, though. There are two reasons. First, one needs to compare mortality rates. Unfortunately, region-specific mortality rates going back to 1800 are not

[2] The Old Northwest corresponds to today's East North Central states: Ohio, Indiana, Illinois, Michigan and Wisconsin.

available. Second, the rate of natural increase also depends on the male-to-female ratio, which was higher in the West. This could offset the effect of higher fertility. Imagine a fertility rate of 1 kid per woman in the East and 2 in the West. Suppose now that there is 1 man per woman in the East and 3 in the West. Then, everything else equal, the rate of natural increase is the same in both locations.

2.2 Land

The territorial expansion of the United States during the nineteenth century was mostly a political and military process. The Louisiana purchase, for instance, was a spectacular acquisition that doubled the size of the country. From the perspective of economic analysis, though, only productive land matters and the process of its acquisition is an investment. At the eve of the nineteenth century, the vast majority of western land had never been used for productive purposes. Settlers moving to the West had to clear, break, drain, irrigate and sometime fence the land before it could be used to produce goods. In doing so they built an important part of the country's capital stock. Gallman (2000, Table 1.12) computed that, in the 1830s, 40 percent of gross investment was accounted for by land improvement – that is clearing, breaking, irrigating, draining and fencing new areas of land to make them productive. Figure 1 displays the stock of improved land. Note how the bulk of improved-land accumulation took place in the West. Note also the magnitude of the increase: one cannot view land as a fixed factor during the nineteenth century in the United States.

Interestingly, the technology for transforming raw land into productive land got better during the century. In other words, a settler in 1900 would improve more acres of land in one day of work than in 1800. Thus, the cost of settling down into the West decreased partly because of technological progress in land-improvement techniques. Primack (1962a,b, 1965, 1969) measured the gain in labor productivity in the various activities contributing to land improvement. Such numbers are unusual to the macroeconomist. For this reason, some details are given below.

2.2.1 Clearing

Consider the clearing and first-breaking of land. What would technological progress, and therefore productivity growth, be like in this activity? Note that it depends on the nature of the soil: clearing an acre of forest or an acre of grassland are two different tasks.

Two methods were common to clear forested areas: the "Swedish" or "Yankee" method and the "Indian" or "Southern" method. The Swedish method consisted mainly in cutting down trees, and then piling and burning the wood. Part of the trees, along a fence line, where cut down but reserved for fencing. Two firings were often needed so that the entire process could take several months. The Indian method consisted in girdling the trees by stripping the bark from a section around it. If done during the winter, the tree would die and start loosing its limbs by the next spring. Eventually, the whole tree would fall. Both methods left the ground studded with stumps. Early frontiersmen would leave the stumps to rot for a few years and then remove them with the aid of basic tools: ax, lever and a yoke of oxen if they had one. Later, mechanical stump-pullers and blasting powder would help them to finish up the land-clearing more quickly. Productivity gain could also have come from specialization. As Primack (1962a) explains, clearing the land usually required much more manpower than individual settlers had available. Groups of settlers would then gather and help each other so that the newcomer did not have to learn and do everything by himself.

Grassland clearing was easier. Yet, the prairie soil required a special kind of plow and a team of four to eight oxen to be first broken. Many settlers did not have the necessary knowledge or material. Hence professionals were commonly hired to break virgin land. According to Primack (1962a), the introduction of an improved breaking plow and its acceptance by farmers, mainly after the civil war, was the main source of productivity increase in grassland clearing.

Table 1 reports data on land-clearing productivity. It took about 32 man-days to clear an acre of forest in 1860 and 1.5 man-days for an acre of grassland. By 1900 these numbers dropped to 26 and 0.5, respectively. One is naturally led to ask what was the proportion of land that was cleared, each period, from different types of coverage. In 1860, 66 percent of the acres cleared were initially under forest cover and 34 percent under grass cover. These numbers changed as more western territories got settled. In 1900, just 36 percent of the land cleared was initially under forest cover, the rest was grassland.

Settlers could choose the type of land they cleared. Hence, the change in labor needed to clear an "average acre" not only captures technological progress, but also the substitution from forest toward prairie. Using a Tornqvist index to correct this effect, one finds that the annual growth rate of productivity in land-clearing, for the period 1860-1900, was 0.6 percent.[3] This figure compares, for instance, with the 0.7 percent annual rate of total factor productivity growth during the same period – see Gallman (2000).

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[3]Let f_0 represents the share of forest in a representative acre at date 0. Let h_0^f and h_0^p represent the labor requirement to clear an acre of forest and prairie at date 0, respectively. The share of forest-clearing in the total cost of clearing is

$$\omega_0 = \frac{f_0 h_0^f}{f_0 h_0^f + (1 - f_0) h_0^p}.$$

The Tornqvist index, for the change in the labor requirement between date 0 and 1, is defined as

$$\ln T = \frac{\omega_0 + \omega_1}{2} \ln\left(\frac{h_0^f}{h_1^f}\right) + \left(1 - \frac{\omega_0 + \omega_1}{2}\right) \ln\left(\frac{h_0^p}{h_1^p}\right).$$

The growth rate of productivity between date 0 and 1 is then $T - 1$.

2.2.2 Fencing

A second component of farm improvement is the construction of fences. Primack (1969) shows that the cost and time required for fencing a farm was far from negligible, and a subject of continuous discontent for farmers. Initially, fences were made out of natural materials adjacent to the site: wood, stones or brushwood. This material was not always abundant depending on the region. Or, it was simply not convenient at all. For example, stone fences were cheap but difficult to build, and even more difficult to move if the enclosed area had to be extended. Consequently, throughout most of the nineteenth century, farmers have sought out better fencing devices. The major cause of productivity gain in fencing was the shift from wood to wire fences. A well known example of a technological innovation can be found here: barbed wire, invented and patented by Joseph F. Glidden in 1874. The effects of such an innovation are quite obvious: barbed wire is light, easier and faster to set up than wood fencing, and withstand fires, floods and high winds. Primack (1969) reports that the fraction of time a farmer devoted to maintaining and repairing fences dropped from four percent in 1850 to 1.3 percent in 1900. Although, strictly speaking, this is not fence-building, it still conveys the idea that fences became easier to handle and a lighter burden on the farmer.

Table 2 reports data on fencing productivity. It took about 0.31 man-days to build a rod of wooden fence in 1850. This number remained unchanged until 1900. Stone fences required two man-days per rod and, here again, this number remained constant until 1900. In 1860, wire fences were made out of straight wire, which required about 0.09 man-days per rod. This requirement dropped to 0.06 man-days in 1900, thanks to the use of barbed wire.[4] The shares of wood, stone and wire fences in total fencing are also reported in the table. A calculation similar to the one carried out in the case of land-clearing reveals that the annual growth rate of productivity in fencing was 0.5 percent between 1860 and 1900.

2.2.3 Draining and Irrigating

The last two activities, drainage and irrigation, did not undergo any productivity gains during the second half of the century. Primack (1962a) argues that, in both cases, the labor requirements for laying one rod of drain or irrigating an acre of land remained constant from 1850 to 1900.

2.3 Hypothesis

What are the mechanism at work behind the Westward Expansion? It is probably fair to say that there exists a standard view on this matter which is as

[4]Source: Primack (1962a, p. 82). The figure for the productivity in wooden fences building is the average of the labor requirement for three types of fences: The “Virginia Rail:” 0.4 man-days, the “Post and Rail:” 0.34 and the “Board:” 0.20. A rod is a measure of length: 16.5 feet. The posts supporting a fence are usually one rod apart.

follows: the abundance of western land, and thus its low price attracted settlers.[5] This is not fully satisfactory for three reasons. First, and to the best of my knowledge, there are no quantitative assessment of the importance of this channel. Second, if raw land was indeed cheap, the cost of transforming it into improved land had to be paid anyways. As mentioned above, this cost was significant enough to represent 40 percent of total investment during the 1830s. Finally, and as explained before, western improved land was not in fixed quantity but rather the result of investment decisions. Viewing productive land as an endogenous variable means that its "abundance" cannot be a driving force. It is, on the contrary, a fact that one has to account for.

Another view emphasizes population growth. Observe Figure 1 and the fact that Eastern land was essentially a fixed factor as of 1800. As population grew, because of natural increase and immigration, eastern wage growth slowed down due to the decreasing returns implied by the fixed stock of land. The existence of the West offered the possibility of increasing the total stock of land, and therefore permitted wage growth to be faster. This view is akin to the "safety valve" hypothesis of Turner (1920).[6]

A third approach emphasizes the transportation revolution which took place during the nineteenth century. O'Rourke and Williamson (1999) and Fishlow (1965, 2000) describe the improvements in technologies and transportation infrastructures. There are two potential effects of the transportation revolution on the Westward Expansion. First, the moving cost for settlers decreased. Second, the cost of shipping goods to and from the West decreased too. Consequently, this reduced the economic isolation of westerners: they could sell their goods on the large markets of the Atlantic coast and purchase consumption and investment goods produced there at lower costs. An important consequence of the decrease in transportation costs is the convergence of regional real wages – see Figure 3.

The view that the transportation revolution could explain the Westward Expansion suggests that other form of technological progress could also do the same. In particular, technological progress in land-improvement techniques and in the production of goods could also have affected the Westward Expansion. For instance, improvement in total factor productivity raised the marginal product of land and labor in the West. However, productivity growth could also have had negative effects. In the East, technological progress allowed wage and consumption growth, despite population growth and decreasing returns. As a result, technological progress slowed down the Westward Expansion by reducing the need to increase the stock of land. The final effect of such changes is ambiguous a priori, and must be investigated quantitatively, using a formal model.

[5] Turner (1920), whose thesis on the American Frontier changed the way scholars envision American history, thought that free land was of great importance. He wrote:

> The existence of an area of free land, its continuous recession, and the advance of American settlement westward explain American development.

[6] The safety valve hypothesis asserts that the opportunities one could find in the West defused social and economic discontent in America.

3 The Model

How do population growth and the various forms of technological progress interact? What are their respective contributions to the Westward Expansion? To answer such questions, this section presents a model incorporating these mechanisms. To clarify the exposition, the first version of the model is static, and it is analyzed through a set of numerical examples. Section 3.2 presents then a dynamic version of the same model, for the purpose of the quantitative exercise conducted in Section 4.

3.1 The Static Model

All activity takes place in a single period of time. There are two locations called East (e) and West (w), and the commodities included are as follows: labor, a consumption good, an intermediate good, and eastern and western land. As will become clear later, the intermediate good serves the purpose of introducing a transportation cost for goods, in addition to a transportation cost payed by households.

Land exists in two states: raw and improved. Only the latter can be used for production. All eastern land is improved exogenously. The stock of improved western land, however, is determined in equilibrium. Specifically, a land-improvement sector, which exists only in the West, hires local workers to transform raw land into improved land. It then sells it to households who, in turn, rent it to the western consumption-good sector.

Each location produces the consumption good with labor, the intermediate good and improved land. However, only the East produces the intermediate good, with labor. Thus, the western consumption-good sector faces a transportation cost to use the intermediate good.

At this point, it is important to lay out the structure of ownership of land and firms. There is a mass p of identical agents, supplying inelastically one unit of time to the market. Each agent is endowed with an equal fraction of the property rights over eastern land and the various firms in the economy. Regardless of their location, households can purchase property rights over western improved land, from the land-improvement sector.

Households choose their location and sector of activity. To summarize, they can work in the consumption-good sector in the East or the West, in the western land-improvement sector or, finally, in the eastern intermediate-good sector. There are no costs associated with changing sector within a location. It is costly, however, to change location. Table 3 summarizes the sectors, their inputs and locations.

3.1.1 Firms

Intermediate Good A constant-returns-to-scale technology is used to produce intermediate goods from labor:

$$x = z_x h_x^e.$$

The variable x denotes the total output of the sector, z_x is an exogenous productivity parameter and h_x^e is eastern labor. Let q_x denote the price of x in terms of the consumption good. The eastern consumption-good sector purchases x at no other cost than q_x. The western consumption-good sector faces a transportation cost, though. For one unit of the intermediate good, the western consumption-good sector faces a price $q_x (1 + \tau_x)$. The difference, $q_x \tau_x$, is lost during shipment – an iceberg cost. Hence, the marginal revenue of the firm is q_x, regardless of the final destination of the good. Its objective is then

$$\max \{q_x x - w^e h_x^e\}, \tag{1}$$

where w^e denotes the eastern real wage rate.

Consumption Good Labor, intermediate goods and improved land are used to produce the consumption good:

$$y^j = z_y \left(h_y^j\right)^{\mu} \left(x^j\right)^{\phi} \left(l^j\right)^{1-\phi-\mu}, \ \mu, \phi \in (0,1).$$

In this expression, the superscript j refers to location ($j = e, w$). The variable h_y^j refers to labor employed in the consumption-good sector in location j. Inputs of the intermediate good and improved land are denoted by x^j and l^j, respectively. The term z_y is the total factor productivity of the sector. It is the same in each location. The objective of the eastern sector is

$$\max \left\{y^e - w^e h_y^e - q_x x^e - r^e l^e\right\} \tag{2}$$

where r^e denotes the rental price of improved land. The western sector solves

$$\max \left\{y^w - w^w h_y^w - q_x (1 + \tau_x) x^w - r^w l^w\right\}. \tag{3}$$

Improved Land Eastern land is entirely improved, therefore the land-improvement sector exists only in the West. There, the total stock of land is fixed and represented by the unit interval. In equilibrium, it is partitioned between improved and raw land. The land-improvement sector decides this partition. At this point, it is important to distinguish between the stock of improved land itself and the production services it delivers to the consumption-good sector. The former is measured by the length of a subset of the unit interval, while the latter is measured in efficiency units. This distinction is used to introduce the notion that land is not homogenous. Specifically, the efficiency units obtained from improving an interval $I \subseteq [0,1]$ is given by $\int_I \Lambda(u)\,du$, where Λ is a density function, assumed to be decreasing. Assume, further, that land is improved from 0 to 1. Thus, a stock of improved land of size $l \in [0,1]$ means that the interval $[0,l] \subseteq [0,1]$ is improved and that the efficiency units of land used in production in the West amount to

$$l^w = \int_0^l \Lambda(u)\,du.$$

The function Λ is given the following particular form

$$\Lambda(u) = 1 - u^{\theta}, \quad \theta > 0.$$

One can interpret the assumptions that Λ is decreasing and that land is improved from 0 to 1 as a shortcut for modeling the fact that the "best" land is improved first. This modeling strategy also ensures an interior solution: land close to 1 delivers close to 0 efficiency units after improvement.

The technology for improving land up to point l requires labor and is represented by

$$l = z_l h_l^w \tag{4}$$

where z_l is a productivity parameter and h_l^w is employment. Let q^w denote the price of an efficiency unit of western land. The optimization problem of the firm is

$$\pi = \max\left\{q^w \int_0^l \Lambda(u)\, du - w^w h_l^w : l = z_l h_l^w\right\}, \tag{5}$$

and the first order condition is

$$q^w \Lambda(l) = \frac{w^w}{z_l}.$$

The left-hand side of this expression is the marginal benefit obtained from the last parcel of land improved, i.e., the efficiency units obtained from this parcel multiplied by their market price. The right-hand side is the marginal cost of improvement. Figure 4 represents the determination of the stock of improved land. Three variables affect it: the western wage rate, the productivity of the land-improvement technology and the price at which efficiency units of land are sold. An increase in the wage rate makes land improvement more costly and, therefore, affects it negatively while productivity plays the opposite role. Finally, an increases in the price of efficiency units of land raise the marginal revenue from land improvement and affects it positively.

3.1.2 Households

Households have preferences represented by $U(c)$, a twice continuously differentiable, increasing and strictly concave function. Imagine that before any activity takes place the population is located in the East. The optimization problem of a household deciding to remain there is given by

$$V^e = \max\left\{U(c^e) : c^e = w^e + \frac{1}{p}(r^e l^e + \pi)\right\}$$

where c^e stands for consumption. As mentioned earlier, households are endowed with property rights over eastern land and the firms. Thus, they receive a fraction $1/p$ of the returns to eastern land, $r^e l^e$, and the profit of the land-improvement sector, π. (The consumption-good and intermediate-good sectors have zero profit in equilibrium.) A household deciding to live in the West faces a similar problem:

$$V^w = \max \left\{ U\left(c^w\right) : c^w = w^w - \tau_h + \frac{1}{p}\left(r^e l^e + \pi\right) \right\}$$

where τ_h is the cost of moving from East to West.

Observe that western improved land does not appear in the budget constraint of households. The reason is the following: suppose a household buys one unit of western improved land at price q^w from the land-improvement sector, and rents it at rate r^w to the consumption-good sector. In equilibrium, households buy land up to the point where $r^w = q^w$. Thus, this operation does not appear in the budget constraint. Western land does, however, deliver income through the profit of the land-improvement sector.

The decision of a household is his location:

$$\max \left\{V^e, V^w\right\}. \tag{6}$$

Denote by h^w and h^e the number of agents who decide to live in the West and in the East, respectively.

3.1.3 Equilibrium

The equilibrium conditions on the eastern and western labor markets are $h_y^e + h_x^e = h^e$ and $h_y^w + h_l^w = h^w$, respectively. The condition on the intermediate goods market is $x^w + x^e = x$. Finally, the equilibrium condition on the consumption good market is

$$h^e c^e + h^w c^w + h^w \tau_h + q^x \tau_x x^w = y^e + y^w.$$

The equilibrium is a list of allocations for firms: $\{h_y^w, x^w, l^w\}$, $\{h_e^w, x^e, l^e\}$, $\{h_l^w\}$, $\{h_x^e\}$; prices: $\{q_x, r^w, r^e, w^w, w^e\}$; and a location choice for households, such that: (i) problems (1), (2), (3), (5) and (6) are solved given prices; (ii) markets clear.

3.1.4 Analysis

One can understand the mechanisms at work in the model through a set of numerical examples. First, one needs to choose parameters (see Table 4) and compute a baseline equilibrium. One can then compute additional equilibria, each of them associated with a change in a single exogenous variable at a time. For instance, the first experiment consists in increasing population from 10 to 15, holding other variables at their baseline level. In the second experiment population is at its baseline level but the transportation cost for households, τ_h, is 0.05 instead of 0.1. Other experiments consist in decreasing the transportation cost for goods, and increasing the productivity parameters, z_y, z_x and z_l. Table 5 shows that each experiment results in an increase of the percentage of population living in the West, and the stock of improved land.

When population increases, as in the first experiment, the demand for consumption goods increases, and the opening of more land in the West is justified to satisfy this demand. Land improvement requires labor, so western population increases. Simultaneously, the larger stock of productive land implies a higher marginal product of labor in the west, and therefore the demand for western workers increases. Observe the drop in wages, due to increased labor supply. Note, finally, that the return to western land increases. On the one hand more land is used for production, so its marginal return tends to decrease. On the other hand more labor and intermediate goods are employed too, raising the marginal product of land. In this particular example, the second effect dominates.

Reduction in transportation costs have expected effects. First, the transportation cost for households dictates the East-West wage gap. As it declines, more households move to the West, reducing the wage rate there and increasing it in the East. Second, a reduction in the transportation cost for intermediate goods induces the western firm to use more of it. This results in an increase in the marginal product schedules for western land and labor and, in turn, raises the demand for labor and improved land.

Productivity growth in each sector also promotes the development of the west. Consider first the consumption-good sector. When z_y increases, the return to western land rises inducing an increase in the stock of improved land. This, in turn raises the demand for labor. Note that, at the same time, an increase in z_y reduces the need for western land, since it makes eastern land more productive. However, this effect does not dominate in this particular example. An increase in z_x tends to reduce the price of intermediate goods, making it cheaper for the western consumption-good sector to use it. Then, as in the case of a drop in the transportation cost, the marginal product, and therefore the demand, for labor and land increase. Finally, an increase in z_l directly promotes land improvement which attracts workers to the West.

3.2 The Dynamic Model

Consider a dynamic version of the model, where all the mechanisms described above are incorporated. Let time be discrete and indexed by $t = 1, \ldots, \infty$. The new ingredients of this version are as follows. First, the land-improvement sector solves a dynamic problem: given that the stock of improved land at the beginning of t is l_t, what should l_{t+1} be? Second, there is a government which owns the initial stock of raw land. It sells it to the land-improvement sector which, as in the static version, improves it and sells it to households. The revenue from selling raw land is transferred to households. This device simplifies the model since one does not have to model the market for raw land. Third, the demography has to be clearly specified. The choice is to represent population as a set of overlapping generations, and to specify an exogenous mechanism for its growth. Within each age group one can find three types of agents. Those who spend their life in a single location, East or West, and those who move from one location to the other, the latter being called “movers.” Fourth, there are economy wide markets for western and eastern improved land. Fifth, the consumption-good and intermediate-good sectors solve static problems, hence they are still described by Equations (1)–(3). Finally, the exogenous productivity variables, evolve in line with $z_{yt} = \gamma_{z_y} z_{y,t-1}$, $z_{xt} = \gamma_{z_x} z_{x,t-1}$ and $z_{lt} = \gamma_{z_l} z_{l,t-1}$ and the transportation costs with $\tau_{ht} = \gamma_{\tau_h} \tau_{h,t-1}$ and $\tau_{xt} = \gamma_{\tau_x} \tau_{x,t-1}$.

3.2.1 Improved-land

The physical description of western land is the same as in the static version of the model. The stock of improved land, however, changes according to

$$l_{t+1} = l_t + z_{lt} h_{lt}^w.$$

In words, each period the stock of improved land increases by a quantity which depends on employment in the land-improvement sector. Hence, the equation above is the dynamic counterpart of Equation (4).

At the beginning of period t, the stock of improved land, l_t, is given. The land-improvement sector decides l_{t+1}, or equivalently h_{lt}^w, and its profit is

$$\pi_t(l_t, l_{t+1}) = q_t^w \int_{l_t}^{l_{t+1}} \Lambda(u) du - w_t^w h_{lt}^w - \int_{l_t}^{l_{t+1}} q_t^r(u) du$$

The first two elements of the profit correspond to the total revenue net of the labor cost. They form the counterpart of the profit function described in Equation (5). The last part is the cost paid by the firm, to the government, for raw land located in the interval $[l_t, l_{t+1}]$. The function $q_t^r(\cdot)$, represents the price of raw land set by the government. Its description is postponed to Section 3.2.3.

The value of the sector at date 1 is

$$\begin{aligned} \max \quad & \pi_1(l_1, l_2) + \sum_{t=2}^{\infty} \left(\prod_{\tau=2}^{t} i_\tau \right)^{-1} \pi_t(l_t, l_{t+1}) \\ \text{s.t.} \quad & l_{t+1} = l_t + z_{lt} h_{lt}^w, \ t \geq 1, \\ & l_1 \text{ given,} \end{aligned} \tag{7}$$

where i_τ is the gross interest rate applied from date $\tau - 1$ to τ. The optimality condition for l_{t+1} is

$$q_t^w \Lambda(l_{t+1}) - \frac{w_t^w}{z_{lt}} - q_t^r(l_{t+1}) = \frac{1}{i_{t+1}} \left(q_{t+1}^w \Lambda(l_{t+1}) - \frac{w_{t+1}^w}{z_{l,t+1}} - q_{t+1}^r(l_{t+1}) \right).$$

The left-hand side of this equation is the marginal profit obtained from improving land up to point l_{t+1}, during period t. The right-hand side is the present value of the marginal profit the firm would realize, if it decided to improve this last "lot" during period $t+1$ when prices and technology are at their period-$t+1$ values. Along an optimal path there should be no profit opportunities from changing the timing of land-improvement. Thus, the two sides of this equation must be equal – this equation is an example of the so-called Hotelling (1931) formula.

3.2.2 Households

Decision Problem Households lives for 2 periods, and there are three types in each age group. First, there are those who spend their life in a single location. They are called "easterners" or "westerners." Second, there are those who change location during their lives. They are called "movers." Preferences are defined over consumption at age 1 and 2, c_1 and c_2, and are represented by

$$\ln(c_1) + \beta \ln(c_2)$$

where β is a discount factor.

The decision to be a mover is made only once in life. Consider the case of an agent who wants to move from East to West. Moving takes place at the beginning of the first period of life and costs τ_{ht} units of the consumption good. If the agent elects to move, he works in the West throughout his life. He differs from a westerner who does not pay the cost of moving, though. As will become clear shortly, moving takes place only from East to West. Thus, in what follows, the term "mover" always refers to this particular direction.

Denote the consumption of an age-a household of type j ($j = e, w, m$) during period t by c^j_{at} and the value function of a type-j household of age 1 at t by V^j_t. One can write:

$$\begin{aligned} V^j_t &= \max \left\{ \ln\left(c^j_{1t}\right) + \beta \ln\left(c^j_{2,t+1}\right) \right\} \\ & \text{s.t.} \quad c^j_{1t} + \frac{c^j_{2,t+1}}{i_{t+1}} = w^j_t + \frac{w^j_{t+1}}{i_{t+1}} + T_t \end{aligned} \tag{8}$$

for $j = e, w$, that is for households who do not move during their lives, and

$$\begin{aligned} V^m_t &= \max \left\{ \ln\left(c^m_{1t}\right) + \beta \ln\left(c^m_{2,t+1}\right) \right\} \\ & \text{s.t.} \quad c^m_{1t} + \frac{c^m_{2,t+1}}{i_{t+1}} = w^w_t + \frac{w^w_{t+1}}{i_{t+1}} + T_t - \tau_{ht} \end{aligned} \tag{9}$$

for $j = m$, that is for movers going from East to West.[7] The term T_t is a transfer received from the government at age-1.

In equilibrium, an age-1 agent in the East at date t must be indifferent between moving to the West or staying. Thus,

$$w^w_t - w^e_t + \frac{w^w_{t+1} - w^e_{t+1}}{i_{t+1}} = \tau_{ht}. \tag{10}$$

In words, the present value of income, net of the moving cost, for an agent just settling down into the West must be the same as for an agent of the same age who stays in the East. Observe that Equation (10) implies that households move only in the westward direction. No household would pay to move from West to East, where the present value of income is lower.

[7]Observe that there is an abuse of notation here, since j does not refer to the location but to the type.

Demography Let p_t denote the size of the age-1 population, so that total population is given by $p_t + p_{t-1}$. Population growth has two sources: natural increase and international immigration. Denote the rates of natural increase in the West and the East by n^w and n^e, respectively. Those rates are location specific to capture the differences observed in the U.S. data – see Yasuba (1962). Denote the rate of international immigration by f. Finally, let p_t^w and p_t^e denote the number of age-1 households located in the West and the East, respectively. Their laws of motion are

$$p_{t+1}^w = (n^w + f)p_t^w + m_t \tag{11}$$

and

$$p_{t+1}^e = (n^e + f)p_t^e - m_t. \tag{12}$$

where m_t is the number of age-1 households deciding to move to the West during period t. Define $\omega_t = p_t^w/p_t$, the proportion of age-1 households located in the West at date t. The law of motion for the age-1 population is then described by

$$\frac{p_{t+1}}{p_t} = n^w\omega_t + n^e(1-\omega_t) + f. \tag{13}$$

3.2.3 Government

How does the government price unimproved land? Here, the choice is to assume that it sets the price that would prevail if unimproved land was privately owned and traded on a market.[8] This dictates the following:

$$q_t^r(u) = \begin{cases} \text{undefined} & \text{for} \quad u < l_t, \\ q_t^w\Lambda(u) - w_t^w/z_{lt} & \text{for} \quad u \in [l_t, l_{t+1}], \\ q_{t+1}^r(u)/i_{t+1} & \text{for} \quad u \geq l_{t+1}. \end{cases} \tag{14}$$

Observe first that, at the beginning of period t, all the land up to l_t has already been improved. Therefore, there is no unimproved land before that point. Second, note that integrating $q_t^r(u)$ over the interval $[l_t, l_{t+1}]$ returns a zero-profit condition. In other words, the difference between the value of improved and unimproved land is the cost of improvement. The last part of the definition of $q_t^r(u)$ is a no-arbitrage condition. Consider a lot, du, that is not improved during period t and remains as such at the beginning of period $t+1$. This is the case for all u satisfying $u \geq l_{t+1}$. What would be the return on such lot, if it was traded on a market? By definition, unimproved land is not productive and therefore the return is $q_{t+1}^r(u)/q_t^r(u)$. In equilibrium, this return must equal the gross interest rate, i_{t+1}.

Under this policy, the first order condition of the land-improvement sector now reads

$$q_t^w\Lambda(l_{t+1}) - \frac{w_t^w}{z_{lt}} = \frac{1}{i_{t+1}}\left(q_{t+1}^w\Lambda(l_{t+1}) - \frac{w_{t+1}^w}{z_{l,t+1}}\right).$$

[8] In an infinitely-lived, representative-agent model, this would mimic the equilibrium that would prevail if unimproved land was privately owned and traded on a market. In an overlapping generations model the equilibrium will be influenced by the timing of transfer payments to the agents or $\{T_t\}_{t=1}^{\infty}$.

Specifically, when virgin land is priced competitively, the decisions of buying land and improving it can be separated. This should not be surprising. As long as the no-arbitrage condition described above holds, the firm cannot reduce the present value of its cost by reallocating its purchases of raw land through time. The value of the land-improvement firm depends only on the timing of land-opening itself.

The revenue collected from selling virgin land is distributed via the transfer T_t to young households. The government's budget constraint is then

$$T_t p_t = \int_{l_t}^{l_{t+1}} q_t^r(u) du. \tag{15}$$

The introduction of the government allows one to avoid modeling explicitly the market for shares of the land-improvement firm. As the second line of Equation (14) makes clear, the profit of the land-improvement sector is captured by the government and redistributed to the age-1 population.

3.2.4 Equilibrium

In equilibrium, the gross rates of return on improved land must be identical across locations. Hence, the gross interest rate is given by

$$i_{t+1} = \frac{r_{t+1}^j + q_{t+1}^j}{q_t^j}, \; j = e, w. \tag{16}$$

Several market-clearing conditions have to hold: first, the eastern labor market must clear:

$$p_t^e + p_{t-1}^e = h_{yt}^e + h_{xt}^e. \tag{17}$$

The left-hand side of this equation represents the total eastern population at date t, that is the labor supply. The demand, on the right-hand side, comes from the consumption-good and the intermediate-good sectors. In the western labor market, a similar condition must hold:

$$p_t^w + p_{t-1}^w = h_{yt}^w + h_{lt}^w. \tag{18}$$

The market for intermediate goods is in equilibrium when

$$x_t^w + x_t^e = x_t \tag{19}$$

and, finally, the market for the consumption good clears when

$$c_t + m_t \tau_{ht} + q_{xt} \tau_{xt} x_t^w = y_t^w + y_t^e. \tag{20}$$

Total consumption, c_t, is given by the sum of consumption of all agents.[9] A competitive equilibrium can now be formally defined.

[9] To be precise, let m_t^j ($j = e, w, m$) be the number of age-1 agents of type j at date t:

$$\begin{aligned} m_t^w &= (n^w + f) p_{t-1}^w \\ m_t^e &= (n^e + f) p_{t-1}^e - m_t \\ m_t^m &= m_t. \end{aligned}$$

Definition 1 *A competitive equilibrium is made of: (i) allocations for households* $\{c_{at}^j\}$ *for* $j = e, w, m$ *and* $a = 1, 2$ *and firms* $\{h_{yt}^w, h_{yt}^e, h_{lt}^w, h_{xt}^e, x_t^w, x_t^e, l^e, l_t^w\}$*; (ii) prices* $\{w_t^j, r_t^j, q_t^j, q_{xt}, i_t, q_t^r(\cdot)\}$ *for* $j = e, w$*; and transfers* $\{T_t\}$ *such that:*

1. *The sequence* $\{h_{xt}^e\}$ *solves* (1) *at all* $t \geq 1$*, given prices;*
2. *The sequence* $\{h_{yt}^e, x_t^e, l^e\}$ *solves* (2) *at all* $t \geq 1$*, given prices;*
3. *The sequence* $\{h_{yt}^w, x_t^w, l_t^w\}$ *solves* (3) *at all* $t \geq 1$*, given prices;*
4. *The sequence* $\{h_{lt}^w\}$ *solves* (7) *at all* $t \geq 1$*, given prices;*
5. *The sequences* $\{c_{at}^w\}$ *and* $\{c_{at}^e\}$ *solve* (8) *at all* $t \geq 1$*, given prices; the sequence* $\{c_{at}^m\}$ *solves* (9)*; and households choose their location optimally, or* (10) *holds;*
6. *Population evolves in line with* (11) *and* (12)*;*
7. *The government prices unimproved land according to* (14)*, and its budget constraint* (15) *holds.*
8. *The equilibrium conditions* (16)-(20) *hold.*

3.2.5 Balanced Growth

In the long-run, land becomes a fixed factor in the West as it is in the East. In other words, the land-improvement sector shuts down and $l_t^w \to \int_0^1 \Lambda(u)du$ as $t \to \infty$. Suppose that the rates of natural increase are the same across regions: $n^w = n^e = n$. Assume, finally, that the transportation costs, τ_{ht} and τ_{xt} are negligible – which is true if $\gamma_{\tau_h}, \gamma_{\tau_x} \in (0, 1)$, that is if transportation costs decrease through time. Then, the economy moves along a balanced growth path which can be described as follows. First population growth is constant and is the same across locations:

$$\gamma_p \equiv p_t^w / p_{t-1}^w = p_t^e / p_{t-1}^e = n + f.$$

Employment in each sector, h_{yt}^w, h_{yt}^e, and h_{xt}^e, grows at rate γ_p too. Consequently, the production of intermediate goods grows at the (gross) rate $\gamma_x = \gamma_p \gamma_{z_x}$, resulting in the production of the consumption good growing at the same rate in each location:

$$\gamma_y = \gamma_{z_y} \gamma_{z_x}^{\phi} \gamma_p^{\phi + \mu}.$$

The wage rate is the same in each location and is growing at rate γ_y / γ_p. The rental rate for land in each location increases at rate γ_y as well as the price of land. This implies, through Equation (16), a constant interest rate along the balanced growth path.

Age-1 westerners are born (or arrived from abroad) in the West (first equation); age-1 easterners are born (or arrived from abroad) in the East but decided not to move (second equation); the number of movers is defined as m_t. Total consumption is

$$c_t = m_t^w c_{1t}^w + m_{t-1}^w c_{2t}^w + m_t^e c_{1t}^e + m_{t-1}^e c_{2t}^e + m_t^m c_{1t}^m + m_{t-1}^m c_{2t}^m.$$

4 Computational Experiment

This section contains two parts. In the first, the parameters are assigned numerical values, and the strategy in this respect is twofold. First, a priori information is used to assign values to the factor shares, (ϕ, μ), the growth rate of productivity in the three sectors $(\gamma_{z_y}, \gamma_{z_x}, \gamma_{z_l})$, the rate of decline of transportation costs $(\gamma_{\tau_h}, \gamma_{\tau_x})$, the initial value of the transportation cost for goods τ_{x1} and the demographic parameters (f, n^w, n^e). The remaining parameters, $(\beta, \theta, l^e, \tau_{h1}, z_{y1}, z_{x1}, z_{l1})$, are chosen to minimize a measure of the distance between the model's predictions and the actual ratio of western to total population, stock of improved land and ratio of western to eastern real wages. This particular procedure implies that the fit of the model to the data is not a finding of the exercise, but rather imposed at the onset.[10] The findings of the quantitative analysis of the model are described in the second part of this section, through a set of counterfactual experiments. Given that the model captures the mechanism that generated the Westward Expansion, as imposed in the first part, how important are each driving force?

4.1 Parameters' Values

4.1.1 Using a Priori Information

Let a model period correspond to 10 years. The exogenous driving forces are productivity variables: $\{z_{yt}, z_{lt}, z_{xt}\}$, and transportation costs: $\{\tau_{ht}, \tau_{xt}\}$. To characterize these trajectories, one needs initial conditions and rates of change. Set the growth rate of z_{yt} to 0.55 percent from 1800 to 1840 and 0.71 percent from 1840 onward, as indicated by Gallman (2000, p. 15). Set the growth rate of z_{xt} to the same values. For productivity in the land-improvement sector, use 0.6 percent annual growth from 1860 onward, as suggested in Section 2. Unfortunately, there are no data on technological progress in land-improvement during the antebellum period. The strategy is then to use labor productivity in agriculture as a proxy. Atack, Bateman, and Parker (2000) report that it grew at an annual rate of 0.3 percent per year from 1800 to 1860. O'Rourke and Williamson (1999, p. 36) mention a 1.5 percent annual rate of decline for transportation costs. Use this number for both transportation costs in the model. Set the initial value of the transportation cost for goods to 50 percent, which corresponds to the price difference for wheat between the east and the midwest at mid-century – see Herrendorf, Schmitz, and Teixeira (2006). The choice of the remaining initial conditions is described later.

Set the labor share to $\mu = 0.6$ and the intermediate goods share to $\phi = 0.2$, implying a land share of 20 percent. These numbers are derived from Gallman (2000, p. 15). Gallman's growth accounting exercise is particularly relevant here because he uses the increasing stock of improved land depicted in Figure 1. This measure is also used below, when fitting the model to the data. Note that, strictly speaking, it is not clear that the intermediate goods share should be the

[10] To the extent, of course, that the fit is acceptable. Figure 5 gives a sense of this.

same as the capital share. In the model, however, the period is ten years so that one can interpret x as a capital good produced in the East, which fully depreciates in 10 years. In the literature, one can find factor shares in a large range of values. For example, Restuccia, Yang, and Zhu (2006) use the agricultural literature to calibrate an agricultural production function. Their land share is 20 percent and their labor share 40 percent, which is significantly less than what is suggested by Gallman's for the aggregate production function. Herrendorf, Schmitz, and Teixeira (2006) also calibrate an agricultural production function and use a labor and land share and of 40 percent both, implying an intermediate goods share of 20 percent. The preference is given to Gallman's figures for the reason mentioned above and because ϕ and μ pertain to the aggregate production function.

Using Haines (2000, p. 153), set the rate of net migration, f, to 5 percent, its average level in the U.S. data for the period 1800-1900. Yasuba (1962) reports information on the birth ratio by state and census year between 1800 and 1860.[11] There is a great deal of variation in this data, but one clear pattern emerges: the larger birth ratio of western women compared with women living in the East. As discussed in Section 2.1, this difference does not imply a higher rate of natural increase in the West. Yet, for the purpose of the quantitative exercise, the rates of natural increase n^w and n^e can differ. This is a conservative choice since it reduces the importance of migration (an endogenous mechanism in the model) in the determination of the geographical distribution of population. How to pick n^w and n^e? First pick n^e. Yasuba indicates that the birth ratios observed in Vermont, Maine, New Hampshire, New York, New Jersey and Pennsylvania are bout 2.0 in 1800. This figure implies a rate of natural increase $n^e = 1 + 2.0/10$. Given $f = 0.05$, $n^e = 1.2$ and the observed ratio of western to total population, one can use Equation (13) to find n^w such that total population would be multiplied by 13.6 in the model (as in the U.S. data) if it replicated exactly the ratio of western to total population from 1800 to 1900. This calculation leads to $n^w = 1.3$.

4.1.2 Minimization

The rest of the parameters are estimated using U.S. data. Before describing this procedure, one must first describe the nature of the computational exercise. Think of starting the economy off at date $t = 1$, that is 1800. At this point, the stock of improved land in the West, l_1, is the result of past investment decisions and, thus, it is given. Set l_1 to the actual ratio of the stock of western improved land in 1800 to its value in 1900: six percent. Normalize the initial old population to one, and assume that there are no old households in the West at date 1. Thus, the initial young population is $p_1 = n^e + f$. From date 1 on, feed the driving variables into the model for a length of 50 periods. Only the first 10 periods are of interest to the quantitative exercise, since they represent the U.S. from 1800 to 1900. Over the remaining 40 periods, gradually reduce the values

[11] The birth ratio is the number of children under 10 years of age, per women aged 16-44.

of n^w and n^e to capture the fact that there has been no discernable differences in regional fertility rates during the twentieth century. Let the remaining driving variables grow at the rates just described.[12] The model economy converges to its balanced growth path in the long-run.

Define $a \equiv (\beta, \theta, l^e, \tau_{h1}, z_{y1}, z_{x1}, z_{l1})$, which is the list of remaining parameters: the discount factor, the curvature parameter (of the density of efficiency units of land), the stock of improved land in the East, and the initial values for the transportation cost for households and productivity paths. For a given a the model generates times series for the ratio of westerners, the stock of improved land and the ratio of western to eastern wages. Namely, define

$$\begin{aligned}\hat{P}_t(a) &= \frac{p_t^w + p_{t-1}^w}{p_t^w + p_{t-1}^w + p_t^e + p_{t-1}^e} \\ \hat{Q}_t(a) &= l_t \\ \hat{R}_t(a) &= w_t^w / w_t^e\end{aligned}$$

Let P_t, Q_t and R_t be the empirical counterparts of $\hat{P}_t$, $\hat{Q}_t$ and $\hat{R}_t$. The sequence Q_t is built by normalizing the stock of western improved land (Figure 1) by its 1900 value. The sequence P_t and R_t are displayed in Figures 1 and 3.

The choice of a is the result of a grid search to solve

$$\min_a \sum_{t\in\mathcal{T}} (\hat{P}_t(a) - P_t)^2 + \sum_{t\in\mathcal{T}} (\hat{Q}_t(a) - Q_t)^2 + \sum_{t\in\mathcal{T}} (\hat{R}_t(a) - R_t)^2 + (i_{1900}(a) - 1.07^{10})^2$$

where $\mathcal{T} \equiv \{1800, 1810, \ldots, 1900\}$.[13] The first and second terms of this distance involve the sum of square differences between the actual and predicted stock of land and ratio of westerners. The third term involves the wage ratio – this is important for the determination of the initial value τ_{h1}. The last term involves i_{1900}, the real interest rate at the end of the period – this is important for the determination of the discount factor β. The value 1.07^{10} correspond to a seven percent annual interest rate.[14] Figure 5 indicates the model's fit to the U.S. data and Table 6 reports the baseline parameters.

4.2 Findings

Under the parameters of Table 6 the model matches the U.S. data reasonably well. One can now use it to ask what are the quantitatively important forces behind the Westward Expansion? To do this, Figures 6 and 7 report the results of six counterfactual experiments. In each experiment a particular driving force is restricted to remain constant at its initial level, while the others are either

[12] The appendix provides the details of the computational procedure.

[13] Note that there is no data for the stock of improved land in 1820 and 1830, and that the data for the wage ratio are for the periods 1830-1880.

[14] In 1900, the model is very close to have reached its steady state where the interest rate is constant. Thus, the correct interest rate target should be a twentieth century rate of return. The choice, here, is to use seven percent, following Cooley and Prescott (1995)'s figure for the second half of the twentieth century.

growing or decreasing as in the baseline case. In the first experiment, for example, the equilibrium path of the economy is computed without any population growth. More precisely, n^w and n^e are set to unity and f to zero. The only forces driving the Westward Expansion are then the technological variables in production and transportation. In the second experiment, population grows as in the baseline case, but there is no productivity growth in the consumption-good sector: $z_{yt} = z_{y1}$ for all t. The third experiment shuts down productivity growth in the land-improvement technology: $z_{lt} = z_{l1}$ for all t. In the fourth experiment, it is the productivity variable in the intermediate goods production which is not growing: $z_{xt} = z_{x1}$ for all t. Finally, experiment five and six correspond to shutting down the decline in transportation costs for households ($\tau_{ht} = \tau_{h1}$) and goods ($\tau_{xt} = \tau_{x1}$), respectively.

The central message from Figures 6 and 7 is that there are two main forces driving the Westward Expansion: the decline in transportation costs (applied to households, that is τ_{ht}) and population growth. The decline in the cost of transportation for households affects mostly the distribution of population, but it has a smaller effect on the accumulation of western land. Population growth, on the other hand, affects mostly the accumulation of land, and has a small effect of the distribution of population.

Specifically, Table 7 reports two measures of the difference between the baseline calibration and the counterfactual experiments. Columns labeled a indicate the sum of squared differences between the baseline trajectories and the counterfactuals (for the ratio of westerner and the stock of improved land.) They confirm the informal discussion of Figures 6 and 7 above: experiments 1 and 5 cause the biggest departures from the baseline case. Columns labeled b indicate the ratio of the 1900 value of each variable to their value in 1900 in the baseline calibration. These columns indicate that, without population growth, the stock of western improved land in 1900 would have been 45 percent of what the baseline model predicts, while the ratio of westerners would have been 97 percent of the baseline prediction. Without technological progress in transportation, the ratio of westerners in 1900 would have been 52 percent of the baseline prediction while the stock of improved land would have accumulated up to 89 percent of the baseline case.

The interpretation of these results follows the same logic as with the static model of Section 2.1. The importance of population growth for the accumulation of western land originates from the fact that eastern land is fixed. As the demand for the consumption good increases because of population growth, decreasing returns in the East makes it more expensive to satisfy this demand. Then, the opening of more western land becomes optimal. The effect of τ_{ht} on the movement of population is straightforward. The fact that it does not affect the development of western land, whereas population growth does, is another indication of the importance of the decreasing returns in the East.

Technology variables such as z_{yt} and z_{xt} have little effects on the accumulation of western land. However, as far as the distribution of population across regions is concerned the growth of z_{yt} plays a noticeable role, although it is quantitatively smaller than the role played by τ_{ht}. Without growth in z_{yt}, that is with no total factor productivity growth, the ratio of western to total population is uniformly below its baseline trajectory, and reaches 92 percent of its baseline value in 1900. To understand, note that in the absence of growth in z_{yt} real wages are decreasing because of population growth. This, on the one hand, tends to push households toward the West where they can develop land to make up for the lack of eastern productivity growth. But, on the other hand, they still face the transportation cost which must be paid out of a decreasing wage. Given the parameters of the model, the second effect dominates. Note, also, that in this experiment land becomes less productive in each location which, a priori affects labor demands in each location. Quantitatively the effect on western labor demand is the largest. The decrease in τ_{xt}, the transportation cost for intermediate goods has a small effect on the distribution of population. Without any reduction in this cost, the ratio of westerners reaches 97 percent of its baseline value in 1900. As far as the accumulation of western land is concerned, this variables plays a negligible role. A surprising result is that the land improvement technology, z_{lt}, has a small effect on the accumulation of productive land, relative to the effect of transportation or population. Without growth in z_{lt}, the stock of improved land in 1900 is 4 percent below its baseline value vis à vis 11 percent when there is no decrease in transportation cost for households, and 55 percent when there is no population growth. One can simply conjecture that this is due to the fact that, relative to other forces, labor productivity in this activity changed little other the course of the nineteenth century. Precisely, z_{lt} is multiplied by a factor 1.5 between 1800 and 1900 while population is multiplied by 13.6 and transportation costs are divided by 6.

Figure 8 indicates the effect of international immigration. The question asked is: if no immigrants from the rest of the world entered the U.S. during the nineteenth century, and if the rate of natural increase remained unchanged, how different would the U.S. be in 1900? Technically, this amounts to computing the equilibrium trajectory of the economy with $f = 0$, but n^w and n^e at their baseline levels. The lesson from this experiment is that international immigration played a small role, quantitatively. Under the assumption that immigrants did not affect the rate of natural increase, population grows by a factor 8.8 between 1800 and 1900 (vis à vis a factor 13.6 in the baseline case). Quantitatively, population growth is still high enough to warrant a significant westward movement.

5 Concluding Remarks

The Westward Expansion during the nineteenth century is one of the processes that shaped the United States as we know it today. In particular, it determined the geographic distribution of population and economic activity. This paper presented an attempt at identifying the quantitatively relevant forces driving this phenomenon. The most important forces are population growth and the decrease in transportation costs. The latter induced the westward migration – without it, only 30 percent of the population would be in the West in 1900, vis à vis 60 percent observed. Population growth is mostly responsible for the investment in productive land – without it less than half of the land accumulated in 1900 would have been accumulated.

Surprisingly, productivity growth shows little effect on the Westward Expansion, relative to the forces just mentioned. In the case of total factor productivity in the consumption good sector, this is due to different effects offsetting each others. On the one hand, the need to move to the West is not that pressing when productivity and wages rise in the East. On the other hand, wage growth makes it cheaper to move. In the case of productivity in the land-improvement sector, the small effect is an unexpected result. One can conjecture that productivity gains in this activity were too small to warrant a bigger contribution.

This paper shows that exogenous natural increase in population were high enough to warrant the Westward Expansion, even if no immigrants came from the rest of the world. Future work could endogenize population growth and investigate its link to the expansion. One can make two observations. First, international immigration from the rest of the world is essentially the same phenomenon as the one studied here, within the United States. Hence, a similar model, calibrated to different data, could shed some light on the pace of international immigration. Second, and regarding natural increase, one faces the challenge of explaining the highest fertility of westerners relative to easterners.

A Computation

This appendix presents the details of the computational procedure. First, as mentioned in the text, the length of the simulation is 50 periods. The reason is that the price of improved land is computed as the present value of future returns, as dictated by iterating Equation (16) forward. Thus, one needs enough periods to reduce the inevitable truncation error in this calculation.

The exogenous path for western fertility, n^w, is $n^w = 1.3$ for periods 1 to 19, $n^w = 1.25$ for periods 20 to 34, $n^w = 1.2$ for periods 35 to 44 and $n^w = 1.05$ for the remaining periods. Similarly, eastern fertility is set at $n^e = 1.2$ for periods 1 to 34, $n^e = 1.15$ for periods 35 to 44 and $n^e = 1.05$ for the rest.

Proceed as follows to compute an equilibrium trajectory. Start with a guess for $\{l_t\}$ the stock of improved land, $\{\omega_t\}$, the proportion of age-1 households located in the West, $\{i_t\}$ the interest rate and $\{x_t\}$ the production of the intermediate-good sector. Use Equation (13) to build a path for total population. (The main text describes the initial population structure and the initial value for the stock of improved land, l_1.) Use the first order and market clearing conditions to build paths for $\{h^w_{yt}, h^e_{yt}, h^w_{lt}, h^e_{xt}, x^w_t, x^e_t\}$ which, in turn, imply trajectories for prices $\{w^w_t, w^e_t, r^w_t, r^e_t, q_{xt}\}$. Iterate Equation (16) forward to obtain q^w_t, q^e_t and q^w_{t+1}. For period t, solve a system of four equations in $(l_{t+1}, i_{t+1}, \omega_t, x_t)$. The equations of this system are: the first order condition of the land-improvement sector, the first order condition of the intermediate-good sector, the market clearing condition for savings and the condition that easterners and movers have to be indifferent. Once this system is solved, move on to solving the same problem for period $t+1$. Convergence is achieved when the final trajectories are close, according to some metric, to the initial guesses.

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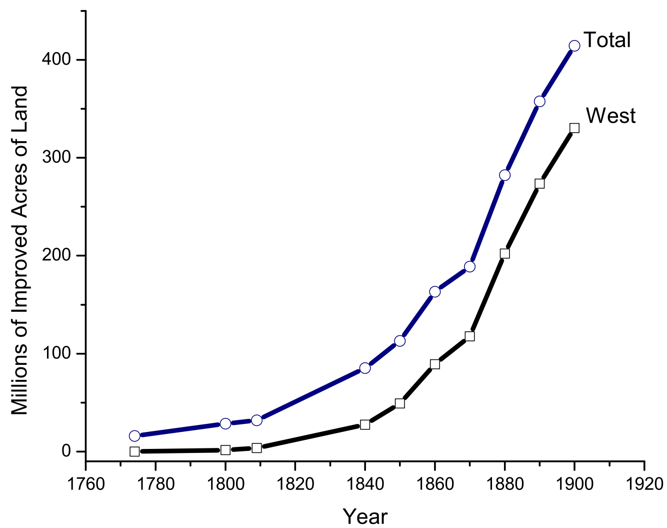

Figure 1: Stock of Improved Land, 1774–1900.

Note – The source of data is Gallman (1986, Table B-5). The "East" is arbitrarily defined as the New-England, Middle-Atlantic and South Atlantic regions. The states in these regions are: Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, New York, New Jersey, Pennsylvania, Delaware, Maryland, District of Columbia, Virginia, West Virginia, North Carolina, South Carolina, Georgia and Florida. The West consists of all other states in the continental U.S.

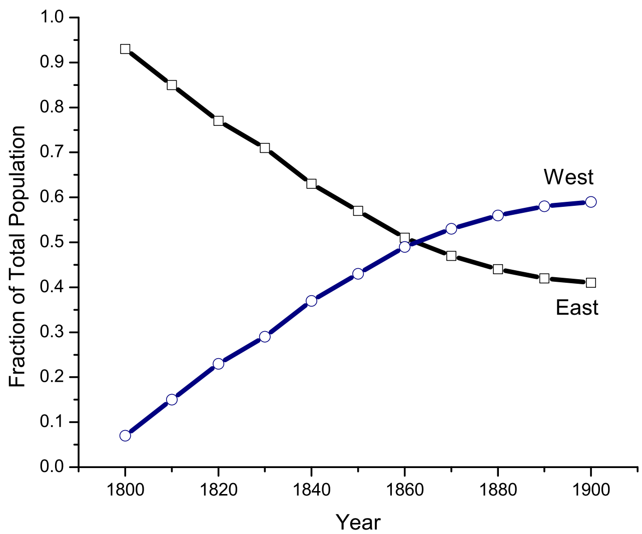

Figure 2: Regional Shares of Total Population, 1790–1910.

Note – The source of data is Mitchell (1998, p. 34).

		1860	1900
(1)	man-days required to clear an acre of forest	32	26
(2)	man-days required to clear an acre of prairie	1.5	0.5
(3)	percent of acre initially under forest	66	36
(4)	percent of acre initially under prairie	34	64

Table 1: Land-Clearing Statistics, 1860 and 1900.

Note – The source of data for lines (1) and (2) is Primack (1962a, p. 28). For lines (3) and (4) it is Primack (1962a, pp. 11-14), the number for 1860 is obtained by averaging the data for the 1850's and the 1860's. Likewise for 1900.

		1860	1900
(1)	man-days required to build a rod of wooden fence	0.31	0.31
(2)	man-days required to build a rod of stone fence	2.0	2.0
(3)	man-days required to build a rod of wire fence	0.09	0.06
(4)	percent of wooden fence	93	0
(5)	percent of stone fence	7	0
(6)	percent of wire fence	0	100

Table 2: Fencing Statistics, 1860 and 1900.

Note – The source of data for lines (1)-(3) is Primack (1962a, p. 82). For lines (4)-(6) it is Primack (1962a, p. 202).

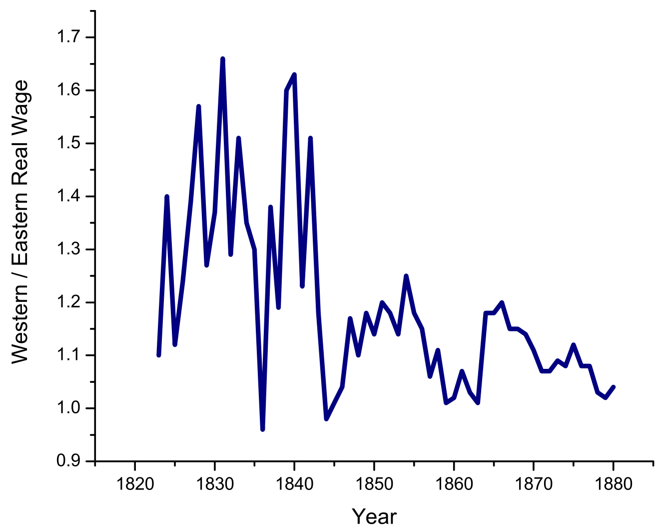

Figure 3: Ratio of Western to Eastern Real Wages, 1823–1880.

Note – The source of data is Coelho and Shepherd (1976) and Margo (2000). Only northern regions, which used free labor throughout the entire period, are considered. The average of New-England and Middle Atlantic's real wages reported by Coelho and Shepherd (1976) are spliced with Margo (2000)'s Northeastern real wages. The average of Eastern North Central and Western North Central real wages from Coelho and Shepherd (1976) are spliced with Margo (2000)'s Midwest real wages.

West		East	
Consumption-good Sector		Consumption-good Sector	
Inputs:	western labor	Inputs:	eastern labor
	intermediate good		intermediate good
	western improved land		eastern improved land
Land-improvement Sector		Intermediate-good Sector	
Input:	western labor	Input:	eastern labor

Table 3: Sectors of Production.

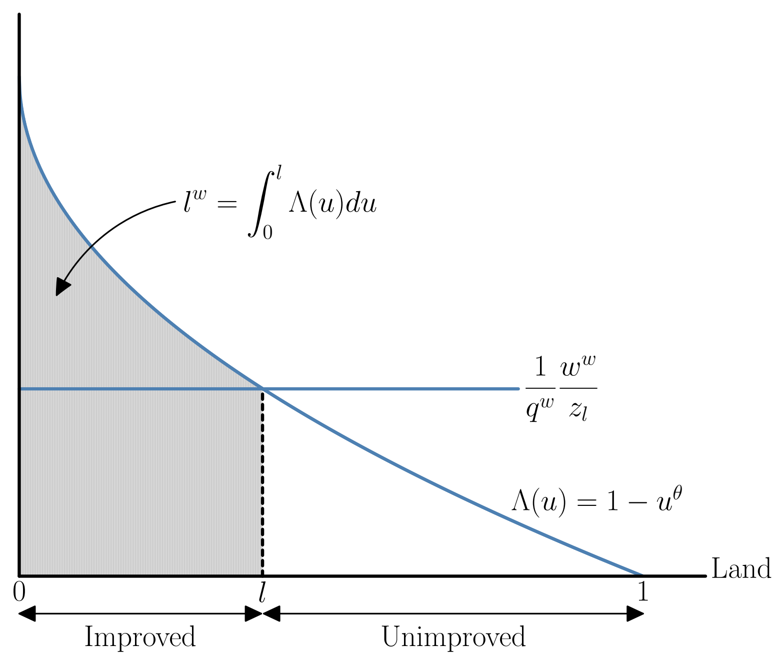

Figure 4: Determination of the Stock of Improved Land.

Technology	$\mu = 0.6$, $\phi = 0.2$, $\theta = 0.5$
	$z_y = 1.0$, $z_l = 1.0$, $z_x = 1.0$
Population	$p = 10$
Eastern land	$l^e = 0.05$
Moving costs	$\tau_x = 0.1$, $\tau_h = 0.1$

Table 4: Static Model – Parameters.

	Ratio of westerners h^w/p	Stock of improved land l	Wage rates (w^w, w^e)	Return to western land r^w
Baseline	**0.17**	**0.18**	**(0.42,0.32)**	**0.73**
$p = 15$	0.21	0.31	$(0.41, 0.31)$	0.92
$\tau_h = 0.05$	0.42	0.38	$(0.39, 0.34)$	1.02
$\tau_x = 0.05$	0.20	0.21	$(0.42, 0.32)$	0.78
$z_y = 1.5$	0.34	0.33	$(0.60, 0.50)$	1.40
$z_x = 1.5$	0.22	0.23	$(0.46, 0.36)$	0.88
$z_l = 1.5$	0.23	0.33	$(0.42, 0.32)$	0.67

Table 5: Static Model – Numerical Examples.

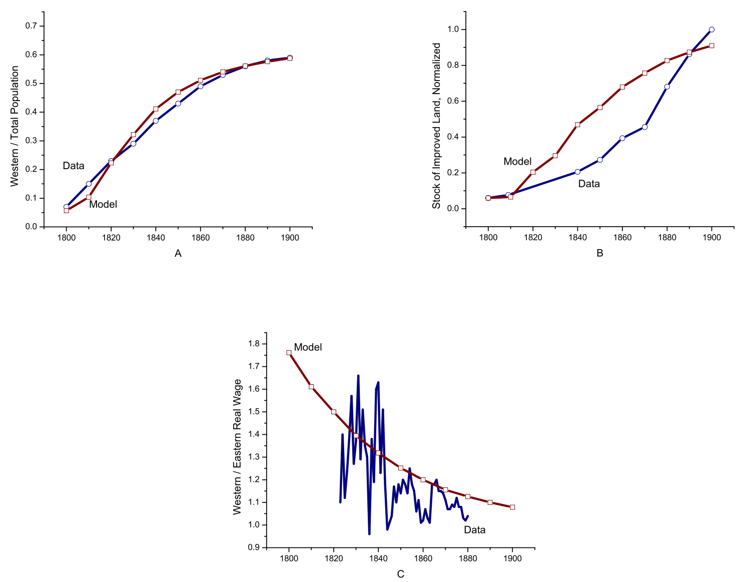

Figure 5: The Model's Fit.

Note – panel A represents the ratio of western to total population, panel B represents the stock of improved land and panel C the ratio of western to eastern real wages.

	Chosen from a priori information	Chosen through Minimization
Preference		$\beta = 0.99$
Technology	$\phi = 0.2$, $\mu = 0.6$	$l^e = 0.01$, $\theta = 0.1$
Demography	$n^w = 1.3$, $n^e = 1.2$, $f = 0.05$	
Driving forces	Growth of z_y: 1.05 and 1.07	$z_{y1} = 2.0$
	Growth of z_x: 1.05 and 1.07	$z_{x1} = 1.0$
	Growth of z_l: 1.03 and 1.06	$z_{l1} = 0.6$
	Growth of τ_h: 0.84	$\tau_{h1} = 0.3$
	Growth of τ_x: 0.84, $\tau_{x1} = 0.5$	

Table 6: Baseline Parameters.

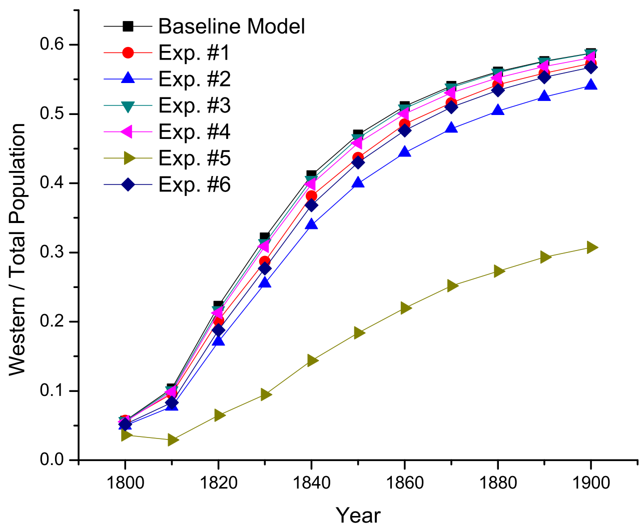

Figure 6: Ratio of Western to Total Population – Counterfactual Experiments.

Note – Experiment 1: No population growth; Experiment 2: No growth in z_{yt}; Experiment 3: No growth in z_{lt}; Experiment 4: No growth in z_{xt}; Experiment 5: No decline in τ_{ht}; Experiment 6: No decline in τ_{xt}.

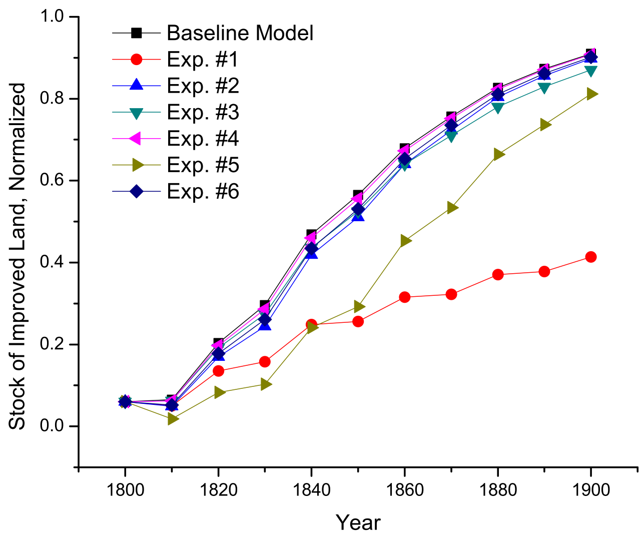

Figure 7: Stock of Western Improved Land – Counterfactual Experiments.

Note – Experiment 1: No population growth; Experiment 2: No growth in z_{yt}; Experiment 3: No growth in z_{lt}; Experiment 4: No growth in z_{xt}; Experiment 5: No decline in τ_{ht}; Experiment 6: No decline in τ_{xt}.

	Ratio of Westerners		Stock of Improved Land	
	a	*b*	*a*	*b*
Baseline	0.000	1.00	0.000	1.00
Experiment 1	0.006	0.97	1.188	0.45
Experiment 2	0.034	0.92	0.013	0.99
Experiment 3	0.000	1.00	0.013	0.96
Experiment 4	0.001	0.99	0.000	1.00
Experiment 5	0.647	0.52	0.336	0.89
Experiment 6	0.011	0.97	0.006	0.99

Table 7: Deviations from the Baseline Case.

Note – *a*: Sum of squared deviations from the baseline case; *b*: Ratio of the 1900 value of the variable to its 1900 value in the baseline.

Experiment 1: No population growth; Experiment 2: No growth in z_{yt}; Experiment 3: No growth in z_{lt}; Experiment 4: No growth in z_{xt}; Experiment 5: No decline in τ_{ht}; Experiment 6: No decline in τ_{xt}.

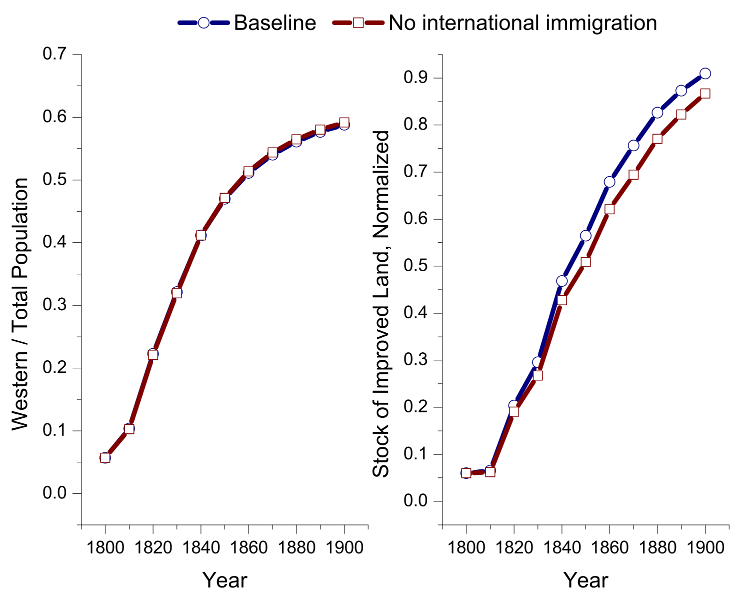

Figure 8: Counterfactual Experiment: The Effect of International Immigration.