A Century of Human Capital and Hours[†]

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Abstract

A person in the United States born in the second half of the nineteenth century completed about 7 years of schooling and spent an average of 58 hours a week working in the market. By contrast, at the end of the twentieth century, people completed close to 14 years of schooling and spent about 40 hours a week working. In the span of 100 years, completed years of schooling doubled and working hours decreased by 31 percent. What explains these trends? We develop a model of human capital and labor supply to quantitatively assess the contribution of exogenous variations in productivity (wage) growth and life expectancy in accounting for the secular increase in educational attainment and the decrease in hours of work. We find that the observed increase in wages and life expectancy account for 87 percent of the increase in years of schooling and 88 percent of the reduction in hours of work. The increase in wages alone accounts for no less than 67 percent of the trend in schooling, and 98 percent of the decline in hours. While changes in life expectancy matter less, their contribution to the increase in schooling is not negligible: no less than 6 percent.

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1 Introduction

Over the course of the nineteenth and twentieth century the United States has witnessed a noticeable increase in various measures of educational attainment. For instance, the mean years of schooling of generations of the second half of the nineteenth century was about 7 while it is close to 14 nowadays. Over the same period of time the lifetime labor supply of a typical worker decreased substantially. For instance, the workweek of a typical worker was around 60 hours in the 1870s and is about 40 hours nowadays. Figures 1 and 2 illustrate these trends. In this paper we develop a model of human capital and labor supply that can broadly capture the secular trends in average years of schooling and hours of work. We use the model to quantitatively assess the importance of productivity growth, reflected as an increase in wages and life expectancy in accounting for these two trends. We find that the observed increase in wages and life expectancy account for 87 percent of the increase in years of schooling and 88 percent of the reduction in hours of work.

The motivation for studying the trends in education and work hours simultaneously is three-fold. First, the trends in education and hours are not specific to the United States but are, instead, common to most developed countries. We can reasonably argue that, over a long period of time, the face of western societies have changed quite dramatically because of reduced hours at most margins (the workweek, increased vacations and retirement length) and the spread of formal education. Second, other authors such as Heckman (1976) and Blinder and Weiss (1976) have emphasized the importance of jointly modeling labor supply and human capital accumulation. Since some dimensions of human capital investment are not observed, models of human capital accumulation are typically restricted by using data on earnings. Recognizing that the accumulation and utilization of human capital have implications for the consumption of leisure time, it is an immediate consequence that we can also use observations about leisure time to bring more discipline to bear on the implications of the model. The study of human capital and labor supply has typically been done in the context of the life cycle (e.g., Blinder and Weiss (1976)). We propose to use the noticeable changes observed over long periods of time as an alternative discipline to these models. Third, our research is connected to the recent literature in macroeconomics on the importance of human capital for understanding inequality across people, time, and countries, e.g. Manuelli and Seshadri (2006), Erosa, Koreshkova, and Restuccia (2010), You (2009), and Guvenen, Kurushku, and Ozkan (2010). By focusing on a time period with substantial changes in labor supply, we find that abstracting from hours of work critically affects the effective returns to human capital investment.

Our model builds on Bils and Klenow (2000). Individuals are homogenous and live for a finite interval of time. They choose how long to stay in school, as well as spending in educational services, and receive human capital from these two inputs. After school they choose how to allocate their time between leisure and work. There are three key features of our model: the human capital production function whereby time and goods are combined to produce human capital, a taste for schooling time, and a non-homotheticity in preferences for the consumption good. For schooling to depend on the level of income, we show that the model should feature both a taste for schooling and a non-homotheticity in preferences. Non-homothetic preferences are central in theories of the structural transformation and development such as Laitner (2000), Kongsamut, Rebelo, and Xie (2001), and Gollin, Parente, and Rogerson (2002); and is a common feature in models of the trend in leisure, e.g. Greenwood and Vandenbroucke (2008). In the model time is continuous and cohorts of constant size (normalized to one) are born at each moment. There are two exogenous variables: life expectancy and wages per unit of human capital. Each cohort faces a different set of values for these exogenous variables and, therefore, makes different educational and labor supply choices. Although we assume that the wage per unit of human capital grows at a constant rate across cohorts, because of the non-homotheticity in preferences, the wage level is relevant for schooling and labor supply decisions of each cohort.

We use our model to compute cohort-specific sequences of labor supply and years of schooling. We proceed as follows. First, we restrict the parameters of the model so that it reproduces the years of schooling and hours of work observed in the data for the 1870 generation. We also impose that the model be consistent with other observations such as the rate of growth in income. We then conduct a set of counterfactual experiments in which we assess the quantitative importance of the main driving forces in the model. We find that wage growth and life expectancy account for 87 percent of the increase in years of schooling and 88 percent of the decline in hours of work between the 1870 and the 1970 cohorts. Among these forces, the growth in wages explains the bulk of the observed changes in both variables: no less than 67 percent of the rise in schooling and almost all (98 percent) of the decline in hours. Life expectancy alone accounts for no less than 6 percent of the rise in schooling and 3 percent of the decline in hours. Wage growth also has level effects since it matters for the wealth of any given generation. Thus, absent wage growth, the time path of schooling would not only be flatter than observed, but also it would be substantially lower, even for the 1870 generation. Similarly, without wage growth, the time path for hours of work would be almost flat and at higher level than observed in the data, even compared with the 1870 generation.

The rest of the paper is organized as follows. We next describe the main two facts on years of schooling and hours of work that are the focus of our analysis. Section 3 presents the model in detail. In Section 4 we calibrate the model and state our main results. Section 5 discusses the results. In Section 6 we conclude.

2 Some Facts

In this section we report the historical trends in average years of schooling and average hours per worker in the United States. Figure 1 reports average years of school completed for males by cohort from Goldin and Katz (2008). The main lesson to take away from Figure 1 is the upward trend. Note also the slowdown observed in the late years of the twentieth century.[1] Hazan (2009, Figure 1) also reports estimates of years of schooling by birth cohort. He finds a substantial increase in the average years of school completed by cohorts over the course of the last 150 years. There are alternative measures of educational attainment, all pointing to a similar picture of a secular increase in education. For example, school enrollment, defined as enrollment in an institution delivering either an elementary, a high-school, or a college degree, has increased from 47 percent of the 5-19 years old population in 1850 to 92 percent in 1990. Likewise, the percentage of persons aged 25 and over with less than 5 years of education decreased from 24 percent in 1910 to less than 2 percent in 2000, while the fraction of persons with at least a bachelor's degree increased from 2.7 to 25.6 percent.[2]

Figure 2 shows the trend in the length of the workweek, i.e., the number of hours worked a week per worker. The main message to take away from Figure 2 is the downward trend and the slowdown of this trend in the second half of the twentieth century. Beside the reduction in weekly hours of work, individuals exploited other margins to reduce their time spent working in the market. Compared with the early 1900s, people work fewer weeks per year and fewer years throughout their life cycle nowadays. Lebergott (1976) reports that 6% of non-farm workers took vacations in 1901, 60% in 1950 and 80% in 1970. Kopecky (2009) reports that in the 1850s a person could expect to spend about 5% of the adult life in retirement. By 2000 this statistic is close to 30%. Hazan (2009) reports estimates of lifetime

[1] Goldin and Katz (2008) show that similar trends are observed for women and across races.

[2] See *Historical Statistics of the United States*, Millennial Edition, series Bc441 and Bc444, and *Digest of Education Statistics*, 2008, Table 8.

hours spent in the labor market per birth cohort. He finds that men born in 1870 spent about 110,000 hours working (total working hours over the lifetime at age 5 by age 79) while men born in 1970 spent less than 74,000 hours: a 33 percent reduction.

3 The Model

Time is continuous. The economy is populated by a set of overlapping generations who live for an interval of length T which is known at the beginning of life. There are perfect credit markets on which individuals can borrow and save at the rate r. Individuals are endowed with one unit of productive time at each moment that can be allocated between leisure and productive activities. Productive activities include the accumulation of human capital while in school and the production of goods. The wage rate per unit of human capital is denoted by w and we assume it grows at the constant rate g.

3.1 Preferences

Preferences are defined over sequences of consumption of goods and leisure time, as well as over the time spent in school. For the sake of simplicity we abstract from life cycle considerations by restricting consumption and leisure time to be constant over an individual's life. Thus, preferences are represented by

$$a_c U(c) + a_\ell V(\ell) + a_s W(s), \tag{1}$$

where c represents consumption, ℓ leisure time, and s time spent in school. We denote the discount factor by ρ so that

$$a_c = \int_0^T e^{-\rho t} dt,$$

and $a_\ell = \alpha a_c$ where α is the weight of leisure in the utility index. The parameter a_s is a positive constant and the functions U, V and W are concave. The function $a_s W$ represents a taste for schooling which follows Bils and Klenow (2000). In the context of our model (as well as in Bils and Klenow's), it is necessary to have $a_s \neq 0$ for the optimal choice of schooling time to depend on the level of income. We will discuss this effect in detail in Section 3.3.[3]

We choose the following functional forms:

$$U(c) = \ln(c - \bar{c}), \; V(\ell) = \ln(\ell), \; W(s) = \ln(s),$$

where $\bar{c} > 0$, can be interpreted as a subsistence level of consumption. As will transpire later the non-homotheticity of preferences for consumption, induced by $\bar{c}$, is critical for the model to generate both the decline in hours of work and the rise in years of schooling as a consequence of income growth. Non-homothetic preferences have a long tradition in development economics and are central in theories of the structural transformation such as Laitner (2000), Kongsamut, Rebelo, and Xie (2001), and Gollin, Parente, and Rogerson (2002). In addition, non-homothetic preferences are a common feature in models of the trend in leisure, see for instance Greenwood and Vandenbroucke (2008). We note the following property of U, which will be useful later:

$$U'(c)c \geq 1 \;\text{ and }\; U'(c)c \to 1 \;\text{ as }\; c \to \infty. \tag{2}$$

[3]There are alternative ways of capturing income effects on schooling, for instance, the Ben-Porath model where the accumulation of human capital depends on the current level of human capital. See the discussion of this alternative modeling in Restuccia and Vandenbroucke (2010).

3.2 Technology

The technology for human capital accumulation is described by

$$H(s, x),$$

where s is time devoted to school and x represents the input of educational services in units of goods. We choose the following specification for H:

$$H(s, x) = x^{\gamma} h(s). \tag{3}$$

and we use

$$h(s) = \exp\left(\frac{\theta}{1-\psi} s^{1-\psi}\right).$$

We choose this functional form to allow comparison with Bils and Klenow (2000). However, we emphasize that this functional form is not critical for our results. The critical restriction on $h(s)$ for the optimal level of s to decrease with the level of w is that $h'(s)/h(s)$ is a decreasing function of s. This restriction is satisfied by a large class of functions.

3.3 Optimization

We assume that the rate of interest is constant and equals the rate of time discount, so the lifetime budget constraint of an individual is given by

$$a_c c + x = w(1-\ell) H(s, x) \int_s^T e^{(g-\rho)t} dt, \tag{4}$$

where w is the wage per unit of human capital prevailing at the beginning of the individual's life. It is convenient to define the discount function $d(s)$ by

$$d(s) = \int_s^T e^{(g-\rho)t} dt.$$

An individual's optimization problem is to choose consumption c, leisure time ℓ, schooling time s, and educational services x, in order to maximize (1) subject to the constraint (4). The first order condition for x implies

$$x = [\gamma w(1-\ell)h(s)d(s)]^{\frac{1}{1-\gamma}},$$

and the optimization problem can then be written as

$$\max_{c,s,\ell} \left\{ a_c U(c) + a_\ell V(\ell) + a_s W(s) : c = \frac{\kappa}{a_c} [w(1-\ell)h(s)d(s)]^{\frac{1}{1-\gamma}} \right\}, \tag{5}$$

where $\kappa = \gamma^{\gamma/(1-\gamma)} - \gamma^{1/(1-\gamma)}$. The first order conditions for ℓ and s are

$$\ell \quad : \quad \frac{a_c}{1-\gamma} U'(c)c - a_\ell V'(\ell)(1-\ell) = 0 \tag{6}$$

$$s \quad : \quad \frac{a_c}{1-\gamma} U'(c)cA(s) + a_s W'(s) = 0, \tag{7}$$

where

$$A(s) = \frac{h'(s)}{h(s)} + \frac{d'(s)}{d(s)}.$$

Consider the optimal choice of schooling and leisure time by individuals who differ on the initial wage w. We summarize the behaviour of our model in the following proposition.

Proposition 1 *The optimal length of schooling and leisure time are increasing in w. That is: $ds/dw > 0$ and $d\ell/dw > 0$.*

Proof 1 *See appendix.*

Proposition 1 shows that the income effect resulting from an increase in w dominates the substitution effect so that individuals purchase more leisure time as they become richer. This property can be interpreted as follows. At low levels of income an individual must work "long" hours to finance the minimum amount of consumption $\bar{c}$. As income increases, the minimum hours required to finance $\bar{c}$ decreases, allowing for the purchase of more leisure time. It is important to note that, because of the non-homotheticity introduced by $\bar{c}$, this mechanism is stronger at low levels of income than at high levels. More specifically, Equation (6) shows that asymptotically –as consumption increases towards infinity– leisure time becomes independent of consumption and of the wage rate w. This is implied by the property of U described in Equation (2). This asymptotic property of the model is consistent with models displaying balanced growth and has been motivated in the literature by the relative constancy of hours during the second half of the twentieth century – see for instance Prescott (1986) and King, Plosser and Rebelo (1988).[4] Following a standard practice in the literature, we will use this asymptotic property of leisure to calibrate the model. Thus, we introduce a notation for the long-run value of leisure time:

$$\tilde{\ell} = \left(1 + \frac{1}{\alpha(1-\gamma)}\right)^{-1}. \tag{8}$$

Equation (7) describes the optimal choice of schooling time. Unlike Equation (6) which is fairly common in models with a consumption-leisure tradeoff, Equation (7) deserves some comment. The marginal benefit of time spent in school has two sources. First, a direct utility benefit $a_s W'(s)$. Second, an increase in income due to the extra units of human capital obtained through schooling. The marginal cost of schooling is due to the foregone

[4]The real business literature often refers to leisure per capita while our motivating data is about hours per worker. McGrattan and Rogerson (2004) show, however, that hours worked by men workers during the 1950-2000 period exhibit little trend. Hours decrease slightly between 1950 and 1970 and increase slightly between 1970 and 2000.

earnings incurred while in school. Both the marginal cost and the pecuniary part of the marginal benefit of schooling are subsumed in $A(s)$. More precisely, $h'(s)/h(s)$ measures the pecuniary part of the marginal benefit of schooling and $d'(s)/d(s)$ measures the marginal cost.

In order to interpret the formal result of Proposition 1, it is useful to write Equation (7) as

$$\frac{c}{1-\gamma}A(s) = -\frac{a_s}{a_c}\frac{W'(s)}{U'(c)},$$

where the right-hand side is the marginal rate of substitution between consumption and schooling time. The left-hand side is the marginal rate of transformation, as dictated by the inter-temporal budget constraint in problem (5). Note that the function $A(s)$ assumes a non-positive value at an optimum so there is a consumption cost to increasing schooling time, i.e. the inter-temporal budget constraint is downward slopping in the (c, s) plan. We show this property formally in the appendix but the economics behind this result deserves some comment. If there was no taste for schooling ($a_s = 0$), the optimal schooling choice would be such that the individual's lifetime income is maximized, which implies that the marginal (pecuniary) benefit and cost of schooling time are equal: $A'(s) = 0$. The individual would then use the optimality condition (6) to allocate income between the purchase of leisure time and consumption. When $a_s > 0$ the marginal benefit of schooling is greater, which induces the individual to acquire more schooling than needed to maximize income. As a result, at an optimum, a decrease in schooling would raise the individual's lifetime income and allow the purchase of more consumption (downward slopping budget constraint). We can represent the optimal schooling choice graphically as in Figure 3 where the optimal schooling level s^* is determined by the tangency point between the indifference curve and budget constraint in the (c, s) plane. Figure 4 shows the effect of an increase in the wage per unit of human capital. The inter-temporal budget constraint shifts up and, given preferences, induces the individual to purchase at the same time more consumption and more schooling. We note, as was the case for leisure, that asymptotically schooling becomes invariant to w since the opportunity cost increases to the point of offsetting the benefits of additional schooling. This can be seen from Equation (7): as $c \to \infty$ and $U'(c)c \to 1$, schooling time becomes independent of consumption and of w as illustrated in Figure 5. We will use this asymptotic property on years of schooling to calibrate the model, thus we introduce the notation $\tilde{s}$ for the asymptotic (long-run) value of years of schooling and is the solution to:

$$\frac{A(\tilde{s})}{1-\gamma} = -\frac{a_s}{a_c} W'(\tilde{s}). \tag{9}$$

As transpires from the previous discussion, the fact that $U'(c)c$ is a decreasing function of c is critical for the time series properties of the model. There are many specifications for U which deliver this property. Our specification for U is guided by the same principle as in modern business cycles and growth theory where long-run increases in wealth have canceling income and substitution effects on labor supply. Specifically, our specification of preferences allows $U'(c)c$ to decrease, but converging asymptotically to a positive constant, namely 1. The convergence of $U'(c)c$ to 1 implies that s converges to a constant in $(0, T)$ as opposed to approaching T and leisure converges to a constant in $(0, 1)$ instead of 1.[5]

4 Quantitative Analysis

The experiment we consider amounts to computing sequences of hours and years of schooling for generations starting in 1870 up to 1970. The exogenous forces in the model are the increase in the wage per unit of human capital w and the increase in life expectancy T.

[5]Convergence of $U'(c)c$ to a positive constant is a consequence of the logarithmic utility specification and the non-homothetic term $\bar{c} > 0$.

These two variables are the only difference across generations in the model.

4.1 Calibration

To perform our quantitative experiment we need to restrict the parameters of the model and the time series for the two exogenous variables. We proceed as follows. First, for the exogenous variables, we assume that they can be represented by an initial condition and a constant growth rate:

$$
\begin{aligned}
w(t) &= w(1870) \times e^{g(t-1870)}, \\
T(t) &= T(1870) \times e^{g_T(t-1870)}.
\end{aligned}
$$

Life expectancy increased substantially between 1870 and 1970. The life expectancy at age 5 was around 55 years for men born in 1870 and close to 71 for men born in 1970. The 16-year increase in life expectancy is an upper bound of the increase in available years for schooling and working since people may change their retirement behaviour. Additional evidence to bear on our choice of T comes from Hazan (2009) who documents a measure of the life-time working years for people born in the cohorts from 1840 to 1930. We extrapolate using these numbers to obtain a measure of the years spent working for the 1970 cohort. The data on completed years of schooling for men by cohort combined with these numbers give the total years at age 5 to be 43 for men born in 1870 and 60 for men born in 1970. This is a 17-year increase. We select g_T such that it produces an 16-year increase in T for the cohort of people between 1870 and 1970. We set the level of T according to the total years in productive activities. However, the implications of the model are not affected by setting the level of T instead to the value of life expectancy. Our choice of parameters is then $T(1870) = 43$ and $g_T = 0.003$.

We adopt the normalization $w(1870) = 1$ and we let the rate of interest (and time discount) be 4 percent: $\rho = 0.04$. Given T and ρ, a sequence of parameters a_c is implied since $a_c = \int_0^T e^{-\rho t}dt$. We also choose $\gamma = 0.1$, following measures of the share of goods and time in the production of human capital. We now turn to the remaining parameters: ψ and θ, the parameters of the human capital technology, g the growth rate of the wage per unit of human capital, and $\bar{c}$, α and a_s, the remaining preference parameters. Bils and Klenow (2000) suggest a range of estimates for ψ between 0 and 0.6. We choose $\psi = 0.3$, the middle of this range, and do sensitivity with respect to this parameter in Section 5.4. We denote by λ the 5×1 vector of parameters left to be determined:

$$\lambda = [\theta, g, \bar{c}, \alpha, a_s]' .$$

We use 5 observations to discipline these parameters. We impose that the model's predictions for years of schooling and hours exactly match the U.S. data for the 1870 generation. That is, we impose that the first generation of the model chooses to stay in school for 7 years and to work 58 hours per week. The number 58 corresponds to the workweek in 1905, which is when the 1870 cohort reaches age 35.[6] We assume that there is a total of $24 - 8$ hours of discretionary time each day, which implies a total of 112 hours per week, so 58 hours translates into $\ell(1870) = 1 - 58/112$. This procedure gives two restrictions. As $w \rightarrow \infty$ our model predicts that hours of work converge to $1 - \tilde{\ell}$ and that years of schooling converge to $\tilde{s}$ if T remains constant. Hence, we impose that $\tilde{\ell} = 1 - 40/112$ and that, if T remained constant at its highest value, years of schooling would be 14.1 in the long-run.[7] This adds two more restrictions. Our choice of 40 and 14.1 as the long-run value for hours and schooling time is motivated by the facts that, first, these are the last values we observe in our sample

[6] Remember that the data on years of schooling are about schooling completed at age 35. See Figure 1.

[7] We note that the procedure of restricting preference and technology parameters to generate asymptotic values in the model is reminiscent of the development literature in setting targets for long-run share of food consumption or the long-run share of employment in agriculture.

and, second, both hours and years of schooling feature a noticeable slowdown in the last years of our sample. The last restriction we impose is that the model reproduces an average increase in income of 2 percent per year.[8]

Formally, our procedure can be described as solving a system of 5 equations in 5 unknowns. For a given λ, we compute years of schooling, labor supply, income and educational expenditures for a sequence of 100 generations born between 1870 and 1970. Our targets for γ are summarized below. Thus, we solve for the zero of the function $F(\lambda)$ defined by

$$F(\lambda) = \begin{bmatrix} s(1870) - 7 \\ \ell(1870) - 0.48 \\ \tilde{s} - 14.1 \\ \tilde{\ell} - 0.64 \\ y(1970)/y(1870) - e^{0.02 \times 100} \end{bmatrix},$$

where y is the period income of a particular generation.[9] Although the system $F(\lambda) = 0$ determines all parameters simultaneously, some parameters are more important for some targets than for others. In particular, the growth rate g has a first order effect on the growth rate of income, and parameters such as $\bar{c}$, θ and a_s also matter in pinning down the initial level of schooling and hours and the long-run level of schooling. Finally, we emphasize that the restriction on the long-run level of hours is independent of the other restrictions. This can be seen from Equation (8) which implies that α must be such that

$$0.64 = \left(1 + \frac{1}{\alpha(1-\gamma)}\right)^{-1}.$$

[8]This increase in income is motivated by data on real Gross Domestic Product per capita from Historical Statistics, see Carter et al (2006, Table Ca-C). However, we note that a similar restriction would follow from using wage data, see for instance Williamson (1995).

[9]For a generation born when the wage per unit of human capital is w, we have $y = we^{gs}H(s,x)(1-\ell)$.

Given that $\gamma = 0.1$ we obtain $\alpha = 2$. It is worth emphasizing this property of the model because it implies that the only preference parameter pertaining to leisure is disciplined by a long-run restriction on hours. Therefore, the initial level of hours imposes discipline on other aspects of the model and, in particular, on the human capital technology. This is the sense in which the historical trend in hours of work imposes additional discipline on the human capital accumulation technology.

4.2 Results

Table 1 reports the parameter values resulting from the calibration of the model. Figures 6 and 7 plot the trend predicted by the model against U.S. data for years of schooling and hours of work. The first two columns of Table 2 summarize the changes in years of schooling and hours in the U.S. and in the baseline version of the model. The first lesson from Table 2 (and Figures 6 and 7) is that the model replicates the bulk of the increase in years of schooling and the decrease in hours. The model predicts that years of schooling increase by 88 percent (from 7 to 13.2) while in the data the increase is 101 percent (from 7 to 14.1). Thus, the model accounts for 88/101=87 percent of the rise in schooling between the 1870 and the 1970 generations. In terms of hours, the model predicts a 27 percent decline (from 58 to 42 hours) while the U.S. data shows a 31 percent drop (from 58 to 40 hours). Hence, the model accounts for 27/31=88 percent of the decline in hours. Given the relatively strong decline in hours, the model also predicts that the lifetime hours of the 1970 cohort decline by 8 percent relative to the lifetime hours of the 1870 cohort, where lifetime hours are computed for a particular cohort as $(1 - \ell)(T - s)$.

A critical parameter for our results is the non-homotheticity parameter $\bar{c}$. This parameter plays a key role to set the level of hours and years of schooling for the 1870 generation in our calibration procedure. It is also critical, in the time series, for the determination of the increase in schooling and the decrease in hours of work. This can be seen from the term $U'(c)c$ in Equations (7) and (6). Our calibration procedure finds that $\bar{c} = 0.26$. In order to gauge how reasonable this number is, we compute the ratio of $\bar{c}$ to income per capita for the 1870 and the 1970 generations. We find that this ratio decreases from 41 percent to 5.5 percent. We compare these numbers to data on final expenditures on food relative to GDP. The data is for the 1996 Benchmark study of the International Comparison Program. For the United States, the share of food is 5.2%. For the average of a set of rich countries (i.e. countries with a GDP per capita no less than 90 percent of that of the U.S.) this share is 5.7%, whereas for a set of poor countries that are between 8 and 10 percent of the U.S. GDP per capita this share is 40.2%. We note that the U.S. in 1870 was about 13 percent of the U.S. in 1970 if growth was around 2% per year. We also note that Maddison (2009) reports that GDP per capita, between 1 and 1500 was between 450 and 771 at constant 1990 dollars. This represents a range of 2 to 4.5 percent of the 1970 GDP per capita. If we interpret the period 1-1500 as one when western Europe was close to subsistence, that is GDP per capita was close to $\bar{c}$, then we conclude that our calibrated value for $\bar{c}$ appears to be within a reasonable range.

Our calibration procedure implies that the model predicts an average growth rate of income of 2 percent per year. This is obtained by setting the growth rate of wages per unit of human capital to 2 percent per year. Hence, the income elasticity of changes in the wage per unit of human capital is $2/2 = 1$. We find a relatively small income elasticity during this period of substantial decline in the amount of work hours.

5 Discussion

5.1 Decomposing the Forces

Our model allows us to quantify the importance of the two driving forces to the rise in educational attainment and the reduction in hours by running counterfactual experiments, and comparing them to the result of the baseline calibration. We compute the path of years of schooling and hours under the counterfactual that each driving force is “shut down.”In the first experiment, we keep w constant at its 1870 value while T increases at the rate g_T as in the baseline. In the second experiment, we keep T constant at its 1870 value while w grows at rate g as in the baseline. The last two columns of Table 2 summarize our results. When w remains constant, years of schooling increase by 5 percent vis à vis 88 percent in the baseline. When T remains constant, years of schooling increase by 59.4 percent. To compute the contribution of w alone to the rise in schooling we note that 5 and 59.4 do not add up to 88. This means that there is a positive interaction between the rise in w and the rise in T. The reason for this is that an increase in life expectancy when the wage rate remains constant represents a smaller increase in wealth than an increase in life expectancy when the wage rate keeps growing over the additional years. The measure of the contribution of changes in w alone depends on how we impute the interaction term between w and T. But this contribution is not less than 67 percent. In terms of the decline in hours, we find that the main driving force is w which accounts for at least 98 percent.

5.2 The Cost of Education

In our model the cost of formal education has two sources: there is a time cost due to the fact that individuals do not work while in school, and there is also a good cost since individuals need to purchase educational services x. In our baseline model we assume that the relative price of these services is constant and equal to one. Recalling that years of schooling implied by the model depart from the data starting around 1920, we note that this is around the time when the high-school movement started. Indeed, Goldin and Katz (2008, chapter 6) place the high school movement between 1910 and 1940. Goldin and Katz emphasize that a significant aspect of the movement was the increased number of educational institutions, both private and public, during this period.

In order to capture this phenomena and gauge its quantitative importance in the context of our model we proceed as follows. We label the relative price of educational services by q, i.e., in order to purchase x units of educational services an individual must give up qx units of consumption. Thus, the individual's inter-temporal budget constraint is of the form

$$a_c c + qx = w(1-\ell)H(s,x)d(s).$$

We then assume that q is constant and equal to 1 until 1920 and that it declines at the rate g_q thereafter. We contemplate different values for g_q. Figure 10 reports the results for years of schooling and hours of work for $g_q = 20$ percent. A substantial reduction in the relative cost of education can bring the implications of the model for years of schooling and hours of work much closer to data.[10] This result illustrates the potential importance of modeling the relative price of educational services in a model of the trend in schooling for the period starting around 1920. We note that our model, augmented with a relative price for educational services, implies that the ratio w/q^{γ} is critical for the individual's decision

[10]Smaller reductions in the price of educational services still bring the implications of the model closer to data. For instance, $g_q = 10$ percent implies the model accounts for 93 percent of the increase in years of schooling and $g_q = 5$ percent implies the model accounts for 91 percent relative to the 87 percent in the baseline.

on years of schooling and hours of work.[11] Hence, one interpretation of the results in this experiment is that both a decline in the relative cost of education (represented by a decline in q) starting around 1920 as well as a faster increase in w resulting from skill-biased technical change starting around 1940 could be explaining the faster increase in years of schooling and decline in hours observed in the data since 1920 relative to the baseline model. Explicitly modeling and measuring these sources of variation over time are important elements that we leave for future research.

5.3 Importance of Labor Supply

An implication of our results is a relatively low income elasticity of changes in the wage per unit of human capital. This elasticity is of interest in a variety of contexts, in particular, the development literature. A critical aspect of our relatively low elasticity arises from the substantial decline in hours of work during the period of analysis. For a given increase in wages, a reduction in hours of work amounts to, other things equal, a reduction in the return to human capital accumulation.

To illustrate the importance of labor supply during the period of analysis, we calibrate a version of the model where $\alpha = 0$ (individuals do not value leisure) and we set a constant labor supply to 49 hours per week, which corresponds to the average observed in the U.S. data between 1870 and 1970 (from 58 to 40). We choose $\bar{c}$, g and θ in order to match the following three targets: (i) 7 years of schooling for the 1870 generation; (ii) 2 percent growth in income per capita; and (iii) a ratio of subsistence consumption to income per capita of 41 percent

[11] The intertemporal budget constraint of an individual, after solving out for educational expenditures, is

$$c = \frac{\kappa}{a_c}\left[\frac{w}{q^{\gamma}}\left(1-\ell\right)h\left(s\right)d\left(s\right)\right]^{\frac{1}{1-\gamma}}$$

for the 1870 generation. The first two targets were part of the baseline calibration. The last target deserves an explanation since it was not part of the baseline calibration strategy. By keeping labor supply constant, we are effectively loosing an observation on labor supply that can be used to restrict a parameter in the human capital technology. In the spirit of Blinder and Weiss (1976), we view this property as a virtue of our initial strategy. The additional target we consider is the ratio of subsistence consumption to income which corresponds to the statistic implied by the baseline calibration. Thus, this exercise is as close as possible to our baseline calibration.

We find that the model without a consumption-leisure tradeoff generates a slightly faster increase in years of schooling: 67 percent versus 63 in the baseline. However, in order to generate the 2 percent increase in income, we only need the wage per unit of human capital to increase at a rate of 1.5 percent per year, as opposed to 2 percent in the baseline calibration. Hence, abstracting from the substantial decline in labor supply yields an income elasticity that is 33 percent larger than in the baseline.

5.4 Sensitivity

Our baseline calibration uses $\psi = 0.3$. To assess the sensitivity of our results to this choice we consider two alternative values, $\psi = 0.4$ and $\psi = 0.2$. For each value of ψ, we recalibrate the model using the same method as described in Section 4.1. The results are displayed in Table 3. The main conclusions from our baseline calibration remain. First, the baseline simulation (when both w and T increase) accounts for close to 90 percent of the increase in schooling and 88 percent of the reduction in hours. Second, the increase in the wage per unit of human capital accounts for most of the change in schooling (more than 70 percent) but the contribution of life expectancy alone is not negligible, ranging from 5 to 18 percent.

We note that the contribution of life expectancy to the rise in schooling is enhanced at low values of ψ.[12] Since in this exercise we recalibrate the model, it is important to identify which parameters are adjusted for the different values of ψ. The main difference with the baseline calibration is in the value of a_s, the weight of schooling time in the utility function, while $\bar{c}$ the subsistence level of consumption and g the growth rate of w remain almost the same.

We now turn to the sensitivity of the result to the choice of $\tilde{s}$ (the long-run value of years of schooling). Based on observations from U.S. data, we selected $\tilde{s} = 14.1$. We explore two alternative values: $\tilde{s} = 14.25$ and $\tilde{s} = 14.5$. We do not consider larger value of $\tilde{s}$ that, given our calibration procedure, would tend to imply negative values for θ. For each alternative value of $\tilde{s}$ we recalibrate the model. The results are displayed in Table 4. The results are somewhat more sensitive to the choice of $\tilde{s}$ than to the choice of ψ, but the overall conclusions from the analysis remain the same: the increase in schooling accounted for by our model is between 88 and 90 percent while the decline in hours accounted for by the model remains 88 percent. In terms of the decomposition, we find again that the main driving force is the increase in the wage rate per unit of human capital: it accounts for no less than 71 percent of the increase in schooling and 98 percent the decline in hours.

Our last sensitivity exercise is with respect to the choice of γ, the share of goods in the human capital technology. We used $\gamma = 0.1$ in our baseline calibration. We recalibrate the model under two alternative values: $\gamma = 0.0$ and $\gamma = 0.2$. Table 5 shows the results. As in the case of ψ, we find that the changes in schooling and hours are not sensitive to the choice of γ. We find that some of the calibrated parameters are different than in the baseline, most noticeably, the rate of growth of w is 2.2 percent when $\gamma = 0$ and 1.8 percent when $\gamma = 0.2$. Thus, the choice of γ critically matters for the measurement of the rate of technical progress.

[12] Bils and Klenow (2000, Equation 12) is an example where $\psi = 0$ and where each additional year of life expectancy results in an additional year of schooling.

6 Conclusions

We developed a model of human capital accumulation and labor supply to quantitatively assess the contribution of exogenous variations in wage growth and life expectancy in accounting for the secular increase in educational attainment and the decrease in hours of work observed between 1870 and 1970. We find that the increase in wages and life expectancy account for 90 percent of the increase in years of schooling and 86 percent of the reduction in hours of work. Moreover, abstracting from the substantial decline in hours of work during this time period critically affects the implied measure of TFP growth in the model. Admittedly, the model does not account for all the increase in years of schooling. Other forces may be at work. A reduction in the cost of acquiring education in the form of higher-education institutions as emphasized by Goldin and Katz (2008) may be important, specially since the departure of the model from the data starts being noticeable after 1920, see Figure 6. We also abstracted from skill biased technical change which may be important after 1940. We plan to incorporate these important features in our future research.

A Proof of Proposition 1

Define $G(\ell) \equiv V'(\ell)(1-\ell)$, $Z(c) \equiv U'(c)c$ and note that $G, Z > 0$ and that $G', Z' < 0$. Define also $C(s,\ell)$ as the right-hand side of the intertemporal budget constraint in problem (5). Note that $C_s = C(s,\ell)A(s)/(1-\gamma)$ and $C_\ell = -C(s,\ell)(1-\ell)^{-1}/(1-\gamma) < 0$.

The first order conditions (6) and (7) can now be expressed as

$$\frac{a_c}{1-\gamma} Z(C(s,\ell)) - a_\ell G(\ell) = 0$$
$$\frac{a_c}{1-\gamma} Z(C(s,\ell))A(s) + a_s W'(s) = 0,$$

First we show that, at an optimum, the function $A(s)$ is negative. To see this observe that, by construction, $A'(s) < 0$. Then let s_{low} be implicitly defined by $A(s_{low}) = 0$. Since $a_c Z(c)/(1-\gamma) > 0$ and $a_s W'(s) > 0$ for any s, it follows that the solution s^* to the first order condition for s is such that $s^* > s_{low}$. The conclusion follows from the fact that A is decreasing.

Implicitly differentiating the first order conditions with respect to s, ℓ and w yields

$$\frac{ds}{dw} = -\frac{\partial C(s,\ell)/\partial w}{\Delta} \text{ and } \frac{d\ell}{dw} = -\frac{\partial C(s,\ell)/\partial w}{\Phi}$$

where

$$\Delta = C_s - \left(C_\ell - (1-\gamma)\frac{a_\ell}{a_c}\frac{G'}{Z'} \right) \frac{\frac{a_c}{1-\gamma} Z A' + a_s W''}{a_\ell A G'} < 0$$

$$\Phi = C_\ell - (1-\gamma)\frac{a_\ell}{a_c}\frac{G'}{Z'} - \frac{a_\ell A G'}{\frac{a_c}{1-\gamma} Z A' + a_s W''} C_s < 0.$$

The signs of Δ and Φ are derived from the properties of the functions C, A, Z, G and W. It is immediate that $\partial C(s,\ell)/\partial w > 0$, therefore $ds/dw > 0$ and $d\ell/dw > 0$. ∎

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Table 1: Calibration

Preferences	$\rho = 0.04$, $\alpha = 2$, $a_s = 11.28$
	$\bar{c} = 0.26$
Technology	$\gamma = 0.1$
	$\theta = 0.004$, $\psi = 0.3$
Productivity	$g = 0.020$, $w(1870) = 1$
Demography	$g_T = 0.003$, $T(1870) = 43$

Table 2: Percentage change in years of schooling, hours, and income

	U.S.	Model		
	Data (1)	Baseline (2)	only T grows	only w grows
Years of school	+101.5	+88.6	+5.0	+59.4
relative to (1)	1.0	0.87		
relative to (2)		1.0	0.06	0.67
Hours	−31.0	−27.4	−0.9	−26.9
relative to (1)	1.0	0.88		
relative to (2)		1.0	0.03	0.98
Income	2.00	2.00	0.00	1.94

Note: The numbers for income are annualized rates of growth.

Table 3: Percentage change in years of schooling, hours, and income under alternative choices for ψ

		U.S.	Model		
		Data (1)	Baseline (2)	only T grows	only w grows
$\psi = 0.4$	Years of school	+101.5	+88.4	+4.7	+60.1
	relative to (1)	1.0	0.87		
	relative to (2)		1.0	0.05	0.68
	Hours	−31.0	−27.4	−0.9	−26.9
	relative to (1)	1.0	0.88		
	relative to (2)		1.0	0.03	0.98
	Income	2.00	2.00	0.00	1.94
$\psi = 0.2$	Years of school	+101.5	+92.7	+16.7	+39.3
	relative to (1)	1.0	0.91		
	relative to (2)		1.0	0.18	0.42
	Hours	−31.0	−27.3	−0.7	−26.8
	relative to (1)	1.0	0.88		
	relative to (2)		1.0	0.03	0.98
	Income	2.00	2.00	0.02	1.83

Note: The numbers for income are annualized rates of growth.

Table 4: Percentage change in years of schooling, hours, and income under alternative choices for $\tilde{s}$

		U.S.	Model		
		Data (1)	Baseline (2)	only T grows	only w grows
$\tilde{s} = 14.25$	Years of school	+101.5	+89.7	+3.5	+64.0
	relative to (1)	1.0	0.88		
	relative to (2)		1.0	0.04	0.71
	Hours	−31.0	−27.4	−1.0	−26.9
	relative to (1)	1.0	0.88		
	relative to (2)		1.0	0.04	0.98
	Income	2.00	2.00	0.00	1.95
$\tilde{s} = 14.5$	Years of school	+101.5	+91.5	+1.2	+71.7
	relative to (1)	1.0	0.90		
	relative to (2)		1.0	0.01	0.78
	Hours	−31.0	−27.4	−1.1	−26.8
	relative to (1)	1.0	0.88		
	relative to (2)		1.0	0.04	0.98
	Income	2.00	2.00	−0.00	1.98

Note: The numbers for income are annualized rates of growth.

Table 5: Percentage change in years of schooling, hours, and income under alternative choices for γ

		U.S.	Model		
		Data (1)	Baseline (2)	only T grows	only w grows
$\gamma = 0.2$	Years of school	+101.5	+88.0	+5.3	+61.6
	relative to (1)	1.0	0.87		
	relative to (2)		1.0	0.06	0.70
	Hours	−31.0	−27.4	−1.3	−26.8
	relative to (1)	1.0	0.88		
	relative to (2)		1.0	0.05	0.98
	Income	2.00	2.00	0.01	1.93
$\gamma = 0.0$	Years of school	+101.5	+89.2	+4.9	+56.8
	relative to (1)	1.0	0.88		
	relative to (2)		1.0	0.05	0.64
	Hours	−31.0	−27.4	−0.5	−27.0
	relative to (1)	1.0	0.88		
	relative to (2)		1.0	0.02	0.98
	Income	2.00	2.00	−0.00	1.95

Note: The reported for income are annualized rates of growth.

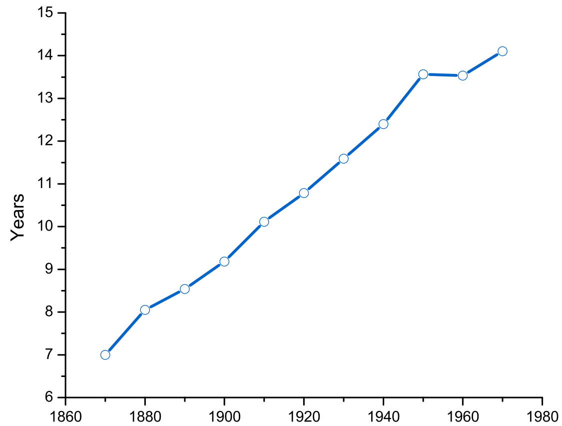

Figure 1: Years of school completed by males at age 35, by birth cohort

Source: The data was kindly provided by Claudia Goldin and Larry Katz. It appears in Goldin and Katz (2008, Figure 1.5).

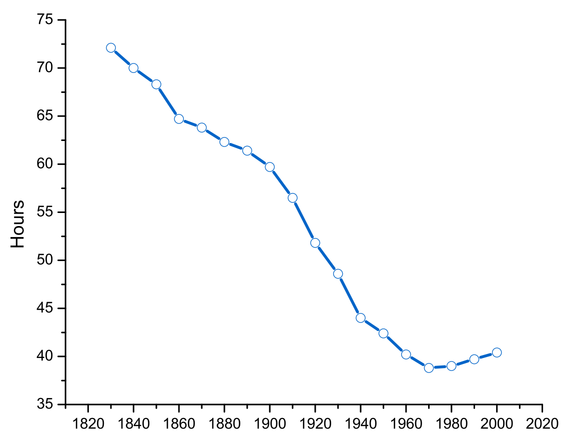

Figure 2: Weekly hours worked per worker

Source: Kendrick (1961, Table A-IV), McGrattan and Rogerson (2004, Table 1) and Whaples (1990, Table 2.1).

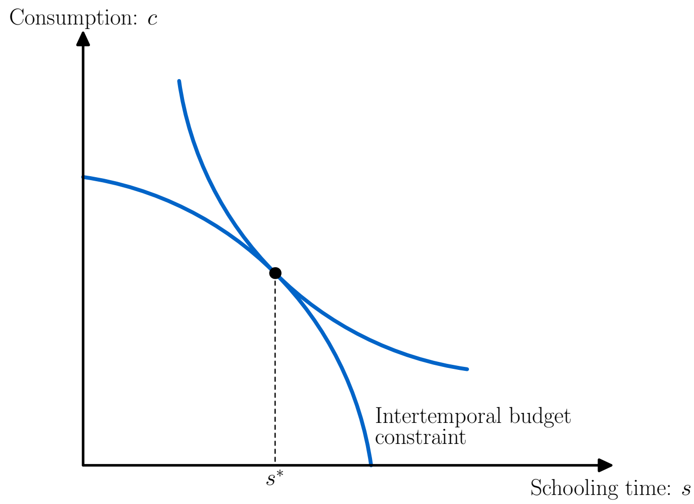

Figure 3: The determination of s^*

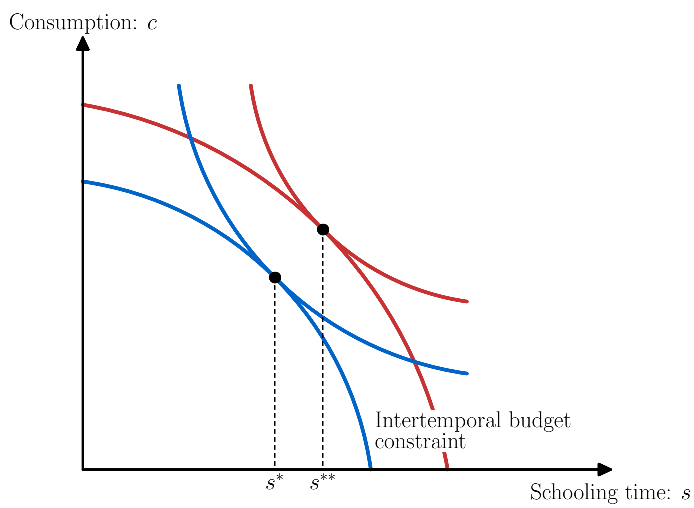

Figure 4: The effect of an increase in the wage per unit of human capital

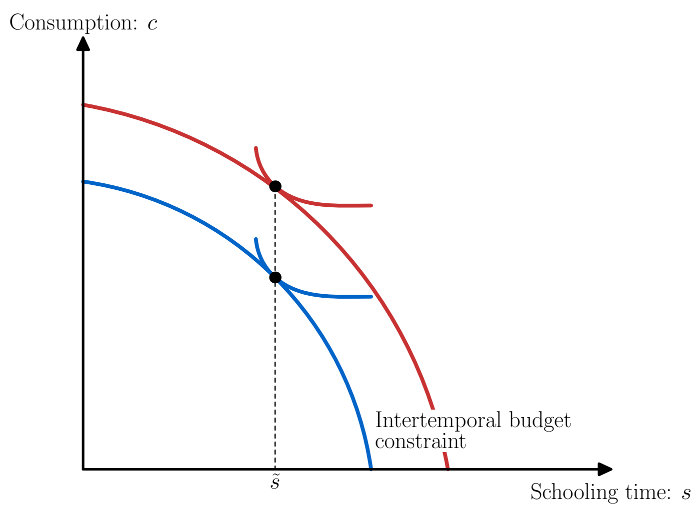

Figure 5: The long-run effect of an increase in the wage per unit of human capital

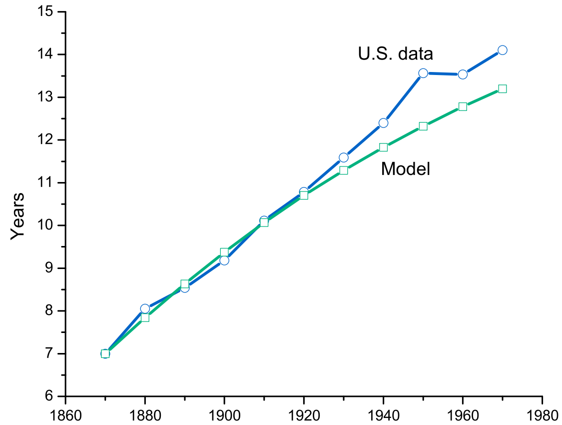

Figure 6: Years of schooling by birth cohort – model and U.S. data

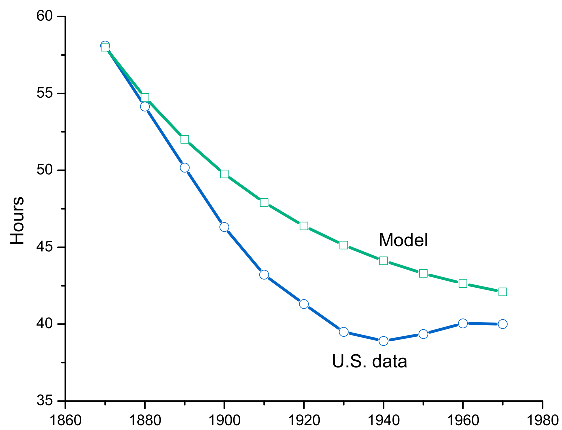

Figure 7: Hours by birth cohort – model and U.S. data

Note: We associate the hours worked of an individual of cohort τ with the workweek observed in $\tau + 35$. For example, the 1900 cohort reaches age 35 in 1935, so we compare the model's prediction for hours of this cohort with the workweek in 1935, which we derive from Figure 2.

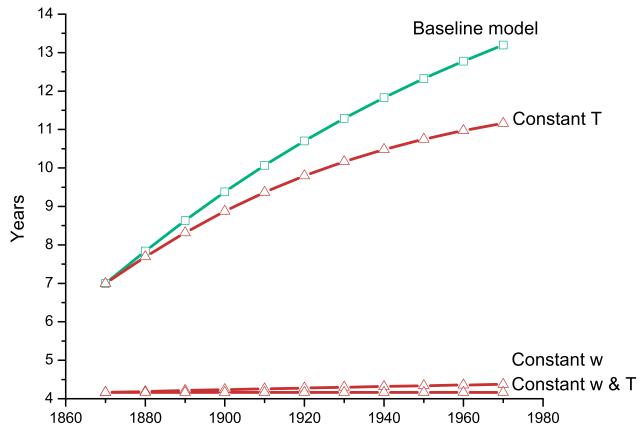

Figure 8: Years of schooling by birth cohort – baseline and counterfactual experiments

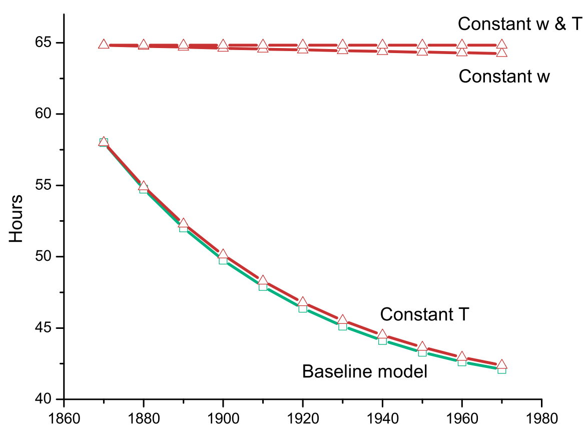

Figure 9: Hours by birth cohort – baseline and counterfactual experiments

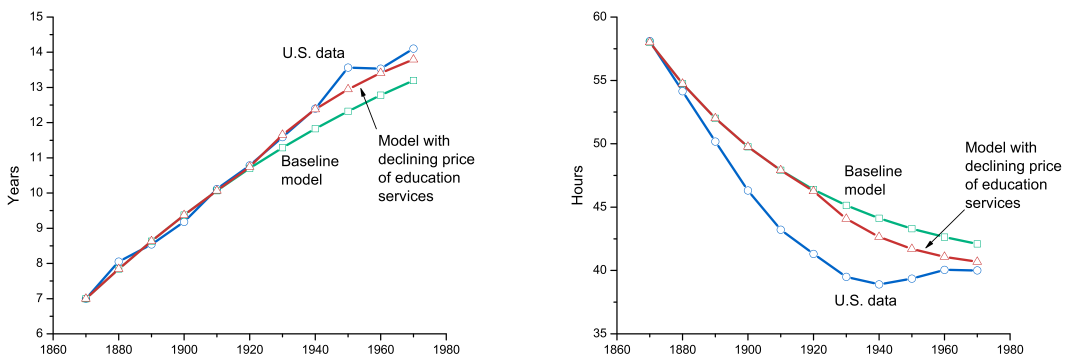

Figure 10: Years of school and hours by birth cohort – U.S. data, baseline model and experiment with declining relative price of educational services