Comparative Advantage and Moonlighting*

Stéphane Auray[†] David L. Fuller[‡] Guillaume Vandenbroucke[§]

August 2021

Abstract

We document three facts: (i) Higher educated workers are more likely to moonlight; (ii) conditional on education, workers with higher wages are less likely to moonlight; and (iii) the prevalence of moonlighting is declining over time for all education groups. We develop an equilibrium model of the labor market to explain these patterns. A dominating income effect explains the negative correlation of moonlighting with productivity in the cross section and the downward trend over time. A higher part-to-full time pay differential for skilled workers (a comparative advantage) explains the positive correlation with education. We provide empirical evidence of the comparative advantage using CPS data. We calibrate the model to 1994 cross-sectional data and assess its ability to reproduce the 2017 data. The driving forces are productivity variables and the proportion of skilled workers. The model accounts for 56% of the moonlighting trend for skilled workers, and 67% for unskilled workers.

JEL codes: E1, J2, J22, J24, O4

Keywords: Macroeconomics, labor supply, multiple jobholders, productivity, full-time job, part-time job, comparative advantage, income effect.

---

*We thank the editor and two anonymous referees for thoughtful comments. We also thank participants at the Brown bag seminar of the Federal Reserve Bank of Saint Louis. S. Auray and D. Fuller thank CEPREMAP and the Chaire "Sécurisation des Parcours Professionnels at Sciences Po and GENES" for financial support. The views expressed in this article are ours and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

†CREST-Ensai and ULCO; Email: stephane.auray@ensai.fr.

‡University of Wisconsin-Oshkosh; Email: fullerd@uwosh.edu

§Research Division, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63166, USA; Email: guillaumevdb@gmail.com.

1 Introduction

Moonlighting, that is when a worker simultaneously holds more than one job, represents an important mechanism for workers to adjust their labor supply. More than 50% of males hold a second job at some point in their working lives (Paxson and Sicherman, 1996). Thus, understanding patterns in moonlighting is important to understand patterns in aggregate hours. Yet, this extensive margin of labor supply adjustment has received little attention. Using data from the Current Population Survey (CPS), we document three facts:

(i) Higher educated workers are more likely to work multiple jobs;

(ii) Conditional on education, workers with higher wages are less likely to work multiple jobs;

(iii) The proportion of multiple jobholders is declining over time for all education groups.

There is an apparent contradiction between fact (i) on the one hand, and facts (ii) and (iii) on the other hand, that is worth noting. Since wages are increasing with education, fact (i) (a cross-section observation) suggests a positive correlation between wages and the prevalence of multiple jobholdings. Fact (ii) (a cross-section observation) indicates a negative correlation, though; fact (iii) (a time-series observation) is also consistent with a negative correlation since wages grow over time.

Labor supply theory implies that hours decrease (increase) with wages when the income (substitution) effect dominates. This adjustment can take place at the extensive margin, that is the number of jobs in this paper, just as well as at the intensive margin in most of the literature (e.g. the business cycle literature). A dominating income effect could thus explain facts (ii)-(iii), but it would imply that higher educated workers are less likely to work multiple jobs. A contradiction to fact (i). Our goal is to provide a quantitative theory of multiple jobholding that is consistent with facts (i), (ii) and (iii).

In Section 3 we develop a static equilibrium model of the labor market with the following features. Workers are heterogeneous along three dimensions: their education (skilled or unskilled), their productivity, and their preference for leisure. There are two types of jobs, full-time and part-time, and workers may choose one of five combinations: one full-time job, one part-time job, two full-time jobs, two part-time jobs and one full- and one part-time job. We focus exclusively on extensive margin adjustments. Thus, hours of work are fixed for each type of job. This assumption is similar to an hours restriction, which Paxson and Sicherman (1996) and Lalé (2019) find to be a reason why multiple jobholders exist. We model the demand for labor via a production technology where the inputs of workers are imperfect substitutes across education (skill v. unskilled) and job type (part- v. full-time).

Two distinct mechanisms allow the model to be consistent with facts (i)-(iii). First, and as noted earlier, a dominating income effect yields the negative correlation between wages and the prevalence of multiple jobholdings, both in the cross-section and in the time series—facts (ii)-(iii).

Second, a comparative advantage of skilled workers in part-time jobs allows the model to be consistent with fact (i). Specifically, if the part-to-full-time wage ratio is higher for skilled than unskilled workers, then the model is consistent with fact (i). The logic is as follows: consider two workers, one skilled and one unskilled, with identical preferences for leisure. Suppose both work one full time job and contemplate a part-time job as a second job. The cost of the second job is the same for both, i.e., the foregone leisure. The benefit differs, however. The skilled worker's marginal utility of consumption is lower (because of higher wages) than that of the unskilled worker. In order to induce the skilled worker to take on the second job, but not the unskilled worker, it must be that the increase in income resulting from the part-time job is larger for the skilled worker than for the unskilled.

We make the following observations about these two mechanisms. First, we estimate Probit models of multiple jobholding and find that conditional on education (and other controls) the most productive workers are the least likely to moonlight. We interpret this as evidence of a dominating income effect and note that it is consistent with evidence of a dominating income effect of wages on labor supply in general: in the long-run decline of hours in the U.S. (McGrattan and Rogerson, 2004; Vandenbroucke, 2009); in cross-country and time series hours data (Restuccia and Vandenbroucke, 2014); and in studies based on micro, cross-sectional hours data (Pijoan-Mas, 2006; Heathcote et al., 2014); finally, Chang et al. (2019) emphasize the potential importance of a dominating income effect to understand the behavior of labor supply over the business cycle. This (non exhaustive) list should convince the reader that our emphasis on a dominating income effect is not at odds with the literature on labor supply. Second, we provide evidence of the comparative advantage of skilled workers in holding a second part-time job in Section 2. Using the Current Population Survey, we show that the change in hourly earnings between 1- and 2-job holders is increasing in education.

Empirically, we proceed as follows. First, we calibrate the model to cross-sectional U.S. data in 1994 because cross-sectional data impose discipline on both the comparative advantage effect (fact (i)) and the income effect (fact (ii)). The list of targeted moments for this calibration includes the proportion of multiple jobholders by education, the college premium, the variance of log-earnings by education and the marginal effect of wages on the proportion of multiple jobholders by education, estimated from our Probit models. The complete list of moments is described in Section 4.

Second, we evaluate the model's ability to reproduce the time series behavior of multiple jobholding. We compute an equilibrium corresponding to 2017 assuming two differences between 1994 and 2017: productivity increases exogenously in a way that is consistent with the observed change in the college premium and overall income growth; and the proportion of skilled workers increases as in the U.S. data. The model accounts for 56% of the decline in moonlighting among skilled workers, and 67% of the decline for unskilled workers.

Our parsimonious static model abstracts from the observation made by Paxson and Sicherman (1996) (among others) that workers move into and out of second jobs frequently. We present, in Appendix D, a dynamic model where second job offers arrive stochastically and second jobs, if accepted, get destroyed stochastically. We draw two conclusions from this exercise.

In both the static and the dynamic models the proportion of multiple job holders is the product of the probability of a second job offer and the probability of acceptance. The static model sets the first probability to one and offers a theory of the probability of acceptance. In the dynamic model the probability of an offer is less than one while the theory of acceptance is identical to that of the static model. Thus, theoretically, the tradeoffs associated with accepting a second job offer are identical in both models. In particular, the role of the comparative advantage is the same in both models.

From an empirical perspective, the dynamic model implies that the downward trend in the proportion of multiple job holders could result from a declining rate of arrival for second jobs offers and/or a declining length for second jobs, and/or a declining acceptance rate. Lalé (2016) concludes that the latter factor "overwhelmingly" explains the trend. This is precisely the focus of our model.

Relation to existing literature

Our paper contributes to the macroeconomic literature on long-run trends and/or country differences in labor markets.[1] A common theme in this literature is the emphasis on some form of extensive margin of labor supply either between home and the market (e.g. Greenwood et al., 2005; Ngai and Pissarides, 2008; Kopecky, 2011; Aguiar et al., 2017); or between schooling, leisure and the market (e.g. Restuccia and Vandenbroucke, 2014); or between sectors (e.g. Rogerson, 2008). We complement this literature by emphasizing another margin of labor supply, i.e., the number of jobs, and by focusing simultaneously on the long-run and the cross-sectional behavior along this margin.

The existing literature on multiple jobholders focuses on understanding why multiple jobholders exist in the first place. An early model can be found in Shishko and Rostker (1976). Kimmel and Smith Conway (2001) documents who moonlights and why. Paxson and Sicherman (1996) hypothesize that multiple jobholding arises from hours restrictions that workers' face, and they focus specifically on the trade-off between job mobility and multiple jobholding. Empirically, they find evidence that hours restrictions indeed drive the phenomenon, and supports a mobility-multiple jobs trade-off. They formulate a dynamic model where workers desiring more hours can either change jobs or add a second job. As we focus on the extensive margin only, our model may be interpreted as imposing a similar hours restriction.

Lalé (2019) also focuses on the micro-determinants of multiple jobholding, and represents an important contribution to the theory of why workers choose multiple jobs instead of increasing hours or changing jobs entirely. He develops a search-theoretic model of the labor market where hours and wages in the primary and secondary job are determined endogenously. While Lalé (2019) focuses primarily on understanding why workers choose multiple jobs instead of more hours, he does use the model to examine the declining trend in the prevalence of multiple jobholders that we also study. He finds this trend results from decreased flows into multiple jobholding (as opposed to shorter duration of the second job). His model attributes most of the declining trend in the prevalence of multiple jobholders

[1] Hirsch et al. (2016) concludes that the rate of multiple jobholders is mostly acyclical. Thus, our paper does not contribute to the literature on the business cycle behavior of hours worked. Overtime hours represent another consideration when examining multiple jobholding. Data on overtime hours paint a very cyclical picture. There is no apparent trend in overtime hours during our period of study, however.

to an increase in the cost of working a second job.

Our paper's contribution to this literature is first to document cross-sectional and time series patterns of multiple jobholding. The emphasis we place on the apparent contradictions between facts (i), (ii) and (iii)) represents the second contribution. Finally, it is the parsimonious model of the labor market we propose to understand the observed patterns of multiple jobholding, and to resolve the apparent contradictions in the data.

2 Data

In this section we describe the data and establish the aforementioned three facts regarding multiple jobholders. The Current Population Survey (CPS) is our primary data set. Specifically we use the Outgoing Rotations Group (ORG). This particular extract of the CPS follows individuals for four months after they enter the survey, they are ignored for eight months, and then interviewed for four more months. The primary advantage of this data set is the availability of earnings information, which is gathered during months four and eight for each individual. In these months individuals are asked questions regarding hours worked and earnings, both overall and in their "usual" job. Multiple jobholders are defined as those workers who had two or more jobs in the reference week of the CPS survey. Data on multiple jobholders are available starting in 1994. While the definition of multiple jobholders includes two or more jobs, in our sample less than 1 percent of all multiple jobholders had more than two jobs; therefore, hereafter we take multiple jobs to refer to an individual working two jobs.[2]

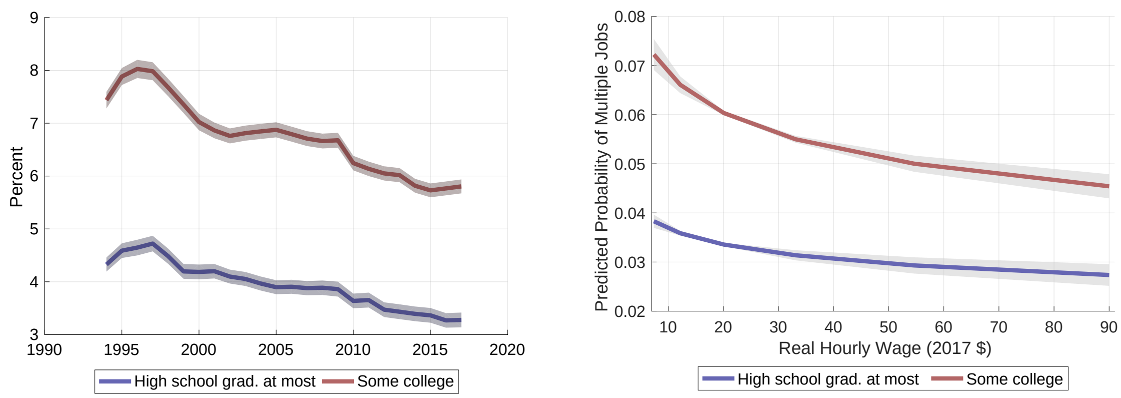
As the CPS represents survey data, the possibility of misreporting of employment status exists. Hirsch and Winters (2016) examine potential bias in the estimates of multiple jobholding produced by the CPS. Specifically, respondents appear more likely to report multiple jobs in the first month in the survey, with declining fractions in each subsequent month, except for a small increase from month 4 to month 5 (see Figure 2 in Hirsch and Winters (2016)).[3] Our sample uses only months 4 and 8 (which contain the earnings information), implying that our estimates of multiple jobholding may understate the true level. Indeed, the overall rate of multiple jobholding in our sample for month 4 respondents is 5.73%, compared to 5.49% for month 8 respondents. This does not affect our analysis below however, as we focus on the individual level determinants of multiple jobholding, and no evidence exists that misreporting is nonrandom in a way that affects our estimates. Moreover, Hirsch and Winters (2016) note that although the declining trend in multiple jobholding is less pronounced if using only month 1 reports, the difference is quite small.

[2] See Appendix C, Figure C.1.

[3] Month 5 is the first month back in after the 8 months following month 4. Thus, this appears to show that the first month in the sample, when the interview is typically conducted in person, produces the most accurate estimates of multiple jobholding.

2.1 Overview

Figure 1 displays the relationship between multiple jobholding, education, and wages. Figure 1A establishes facts (i) and (iii). Those with more education are more likely to work multiple jobs, and regardless of education, the percentage of multiple jobholders exhibits a declining trend.[4] This pattern holds regardless of how coarsely we define the education groups. For example, consider the maximum number of education groups available in the data. In 2015, among workers that did not graduate from high school, 2.1 percent had more than one job. For workers with a high school degree, 3.4 percent were multiple jobholders. For high school graduates that received up to 4 years of college education, the same percentage was 5.1. Finally, 6.3 percent of workers with an advanced college degree had more than one job. This relative ranking is robust across all years in the data.



A – % with two jobs, by education

B – Predicted probability by real wage and education

Figure 1: Relationship between multiple jobholding, education, and real wages

Note: The shaded areas indicate 95% confidence intervals.
Source: Current Population Survey.

There is an apparent contradiction between facts (i) and (iii) represented in Figure 1A. On the one hand workers with higher education and, thus, higher wages (the college premium is never less than 1.5 between 1994 and 2017), are more likely to hold two jobs. On the other hand, as wages increase over time there are fewer multiple jobholders in each education group. The cross-sectional comparison points to a positive correlation between wages and the prevalence of multiple jobholding, the time series points to a negative correlation.

There is also an apparent contradiction between facts (i) and (ii) represented in Figure 1B. This figure plots the probability that a worker holds multiple jobs by real wages, conditional on education. We estimate this probability from Probit models 2A and 2B described in Table 1 (see Section 2.2). On the one hand, workers with higher education are more likely to hold two jobs. On the other hand,

[4]Figure C.2 of Appendix C shows that the downward trend in the proportion of multiple jobholders is true for both men and women, albeit it is more pronounced for men.

conditional on education, workers are less likely to hold two jobs when their wages increase. Again, one comparison points to a positive correlation between wages and the prevalence of multiple jobholding, while the other points to a negative correlation.

Before providing additional details on our characterization of facts (i)-(iii), we briefly describe some other features of multiple jobholding used in our model and quantitative analysis. Figure 2 shows hours worked by type of worker and education. The average single jobholder works slightly less than 40 hours per week, while the average multiple jobholder works around 10 additional hours each week. Note that these figures do not trend over time. We use these observations to justify several features of our model in Section 3. First, we do not model the intensive margin of labor supply. That is, we do not allow workers in our model to adjust their hours by any other means than by adjusting the number of jobs they work. In this, we appeal to the existing literature that finds restrictions limiting a worker's ability to adjust hours in a given job are a key reason for moonlighting (see Paxson and Sicherman (1996) and Lalé (2019)). Second, we assume that there are two types of jobs: a "full-time" job requiring 40 hours of work, and a "part-time" job requiring 10 hours of work. This assumption follows Figure 2, which indicates that multiple jobholders do not work twice as many hours as single jobholders. We adopt these numbers so that the average multiple jobholders in our model works the same additional hours as the average multiple jobholders in the U.S. data.[5]

Multiple jobholders can either work two part-time jobs (PP), two full-time jobs (FF) or one part- and one full-time job (FP). The distribution of multiple jobholders across these categories is noticeably stable over time as shown in Figure 3. Note that FP-workers are the most prevalent, and FF-workers are the least prevalent. This is true for both education categories. We allow workers to choose among these different work arrangements in our model (in addition to the possibility of working one full-time job or one part-time job).[6]

2.2 Cross-sectional Features of Multiple Jobholding

To further explore facts (i) and (ii) we estimate a Probit model with a (0/1) indicator variable for multiple jobholding as the dependent variable. That is, we estimate the following model:

$$\Pr(\text{mjh} = 1) = \Phi(\gamma + \lambda \ln(w) + \beta \mathbf{X} + \epsilon) \tag{1}$$

where Φ denotes the CDF of the normal distribution, w is real hourly earnings on the usual job, including overtime, tips, and commissions, and $\mathbf{X}$ is a vector of covariates. Table 1 presents the results of several specifications.

Model 1 in Table 1 is the most parsimonious specification where multiple jobholding depends on real

[5]Our choice of 10 masks some heterogeneity since a number of multiple jobholders work flexible hours on their main and/or second job. Kimmel and Smith Conway (2001) find that most multiple jobholders work fulltime on their primary jobs and 15 to 20 hours per week on their second jobs.

[6]The lines in Figure 3 do not add up to 100 percent of multiple jobholders because some workers with multiple jobs have more than two jobs or have jobs with variable hours that cannot be classified as either part- or full-time.

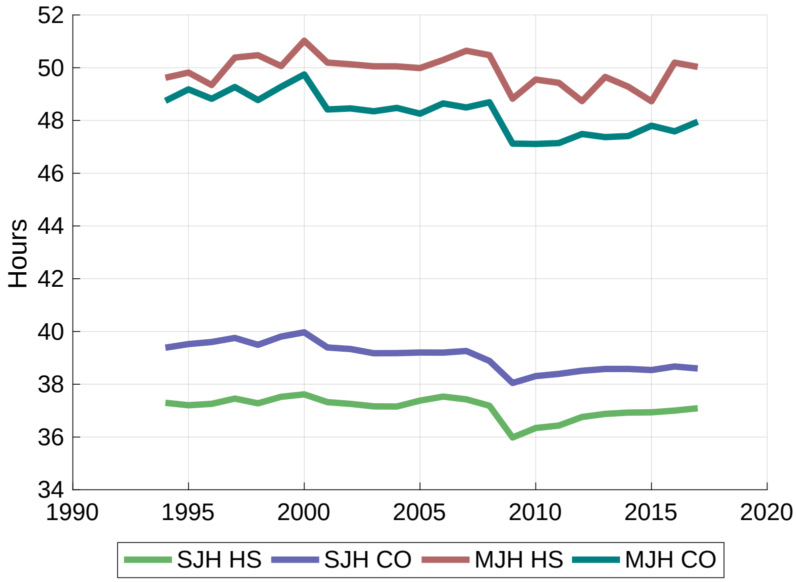
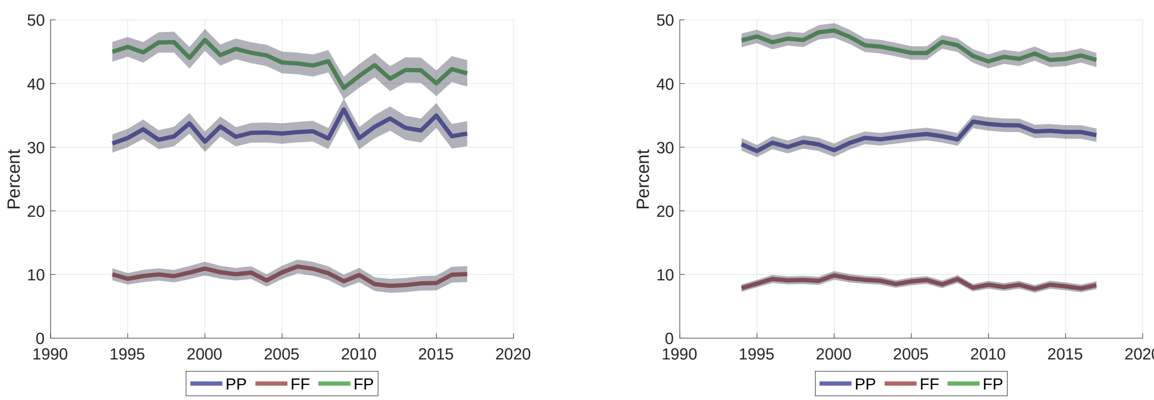


Figure 2: Weekly hours worked

Note: "SJH" means single jobholder and "MJH" means multiple jobholder. "HS" means at most high school graduate and "CO" means some college education.
Source: Current Population Survey.



A - High school graduate at most

B - Some college

Figure 3: Proportion of multiple jobholders by type of jobs and education

Note: The shaded areas indicate 95% confidence intervals.
Source: Current Population Survey.

Table 1: Probit Model Multiple Jobholding Dependent Variable

Variables	Model 1	Model 2A (HS)	Model 2B (CO)	Model 3
ln wage	-0.0134***	-0.0601***	-0.0927***	-0.0444*
	(0.00176)	(0.0104)	(0.00994)	(0.0258)
Female		-0.0475***	-0.0583***	-0.0622***
		(0.0107)	(0.00802)	(0.00777)
Number of children		0.0104***	0.0146***	0.0123***
		(0.00373)	(0.00264)	(0.00214)
Married		-0.0713***	-0.103***	-0.103***
		(0.0120)	(0.00707)	(0.00667)
Hrs. Vary		-0.150***	-0.103***	-0.120***
		(0.0151)	(0.0119)	(0.0104)
Age (Older than 60 reference group)				
Age 20-29		0.00528	-0.00747	0.00300
		(0.0168)	(0.0130)	(0.0110)
Age 30-39		0.0985***	0.0272***	0.0537***
		(0.0160)	(0.00881)	(0.00824)
Age 40-49		0.185***	0.0808***	0.120***
		(0.0124)	(0.00912)	(0.00670)
Age 50-59		0.143***	0.0896***	0.111***
		(0.0125)	(0.00906)	(0.00674)
Race (White reference group)				
Black		-0.0670***	-0.0489***	-0.0480***
		(0.0244)	(0.0149)	(0.0154)
Hispanic		-0.232***	-0.137***	-0.153***
		(0.0191)	(0.0140)	(0.0139)
Other		-0.213***	-0.163***	-0.179***
		(0.0381)	(0.0164)	(0.0217)
Education-Log wage Interactions (Less than HS reference group)				
H.S.#Logwage				-0.0281
				(0.0255)
Some college#Logwage				-0.0533**
				(0.0265)
College#Logwage				-0.141***
				(0.0269)
Advanced#Logwage				-0.0565**
				(0.0260)
Education (Less than HS reference group)				
HS				0.255***
				(0.0693)
Some college				0.510***
				(0.0741)
College				0.832***
				(0.0762)
Advanced				0.677***
				(0.0745)
Constant	-1.554***	-1.429***	-0.946***	-1.521***
	(0.00534)	(0.0351)	(0.0293)	(0.0743)
Observations	3,846,092	732,954	1,263,573	1,996,527
State FE	No	Yes	Yes	Yes
Year FE	No	Yes	Yes	Yes

Standard errors (clustered at the state level) in parentheses
*** p<0.01, ** p<0.05, * p<0.1

wages only. Multiple jobholding and real wages are negatively correlated.

We estimate additional specifications to assess the robustness of this finding. Our baseline specifications are models 2A and 2B in Table 1. For these models we estimate two separate Probit specifications, one for workers with at most a high school education (2A), and one for workers with some college education (2B). In both models we allow for state and year fixed effects as well as demographic controls such as sex, age, race, marital status and the number of children. The variable "Hrs. Vary" is an indicator variable for workers whose hours typically vary on their usual job. The effect of real wages is negative and significant in both models (fact (ii)) and the prevalence of multiple jobholding is higher among workers with some college education, as can be seen from the constant (fact (i)). Female and married workers are less likely to hold multiple jobs. Workers with more children are more likely and non-white workers are less likely to hold multiple jobs. Agewise, the most likely multiple jobholders are middle-aged workers. The probabilities shown in Figure 1B are implied by models 2A and 2B; and in Section 4 we use the slopes of models 2A and 2B with respect to the log of real wages as targets for calibrating our model.

In Model 3 of Table 1 we estimate a specification where we consider five education groups instead of two: less than high-school, high-school, some college, completed college (4 years) and advanced. We allow education to have both a level and a slope effect via an interaction with the real wage. Overall, the prevalence of multiple jobholding is increasing in education, although the coefficient for workers with advanced degrees (0.677) is less than the coefficient for workers with a completed college degree (0.832). Slope coefficients are negative and tend to decrease with education. The strongest coefficient is that of workers with a completed college degree. Demographics such as age, sex, race, marital status and the number of children operate in the same direction as in models 2A and 2B. Overall the lesson from Model 3 is similar to that of Models 2A and 2B. Thus, restricting our analysis to two education groups does not change our characterization of the relationship between education and multiple jobholding, while allowing for a simpler model.

In the models 1-3, the multiple jobs indicator variable is the dependent variable, and thus we do not distinguish between two part-time jobs or a full-time/part-time choice. To verify the robustness of our specifications to alternative combinations of employment choices, in Appendix C we present two versions of Model 3 estimated separately for workers with either a full-time or a part-time main job. Our results are robust and similar to the results described in Table 1.

2.3 Evidence of comparative advantage

As we discussed in Section 1, our model implies that skilled workers must have a comparative advantage in taking a second job in order to explain the higher prevalence of multiple jobholders among them. Specifically, the gain from a second job must be increasing in education. In this section, we provide evidence of this mechanism. We estimate two earnings equations: one for single and one for multiple jobholders, and use these estimates to compute average hourly earnings of workers with one and multiple jobs by education. We use Heckman's (1976) sample correction to account for the fact that selection into the group of multiple or single jobholders is not random:

$$
\begin{aligned}
\ln(e_j) &= \theta_j Z + \nu, \\
\Pr(\text{nb of jobs} = j) &= \Phi(\gamma_j + \beta_j X + \epsilon > 0).
\end{aligned}
$$

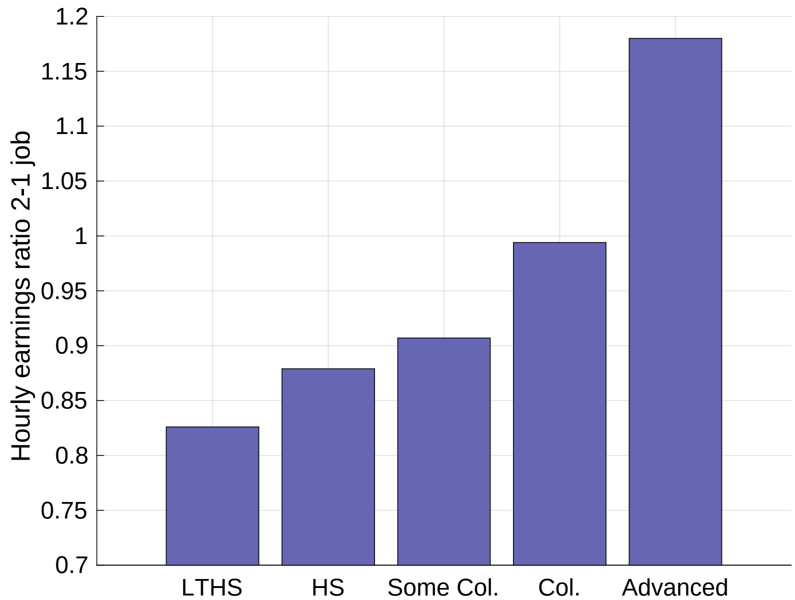


Figure 4: Hourly earnings difference between 1- and 2-jobholders

Source: Current Population Survey and authors' calculations

We estimate this model for single jobholders ($j = 1$) and multiple jobholders ($j = 2$). The variables ϵ and ν are jointly normally distributed with mean zero. The variable e_j indicates *total* hourly earnings on all jobs and Z contains the worker's age (and its square), indicator functions for education, sex and race as well as state, year and occupation fixed effects. The selection equation follows the models we estimated in Table 1.

Figure 4 reports the ratio of *total* hourly earnings (implied by our estimated models) between workers with multiple jobs and workers with one job, by education. The lesson from Figure 4 is that this ratio increases with education and, thus, that workers with the highest education have the most to gain from working multiple jobs. We interpret this as evidence of the comparative advantage mechanism that our model implies is necessary to explain fact (i).

It is important to note that our calculation does not directly compare the earnings on a worker's first and second job. Instead, it compares *total* hourly earnings between single jobholders and multiple jobholders. Figure 4 reveals that *total* hourly earnings of multiple jobholders tend to be lower than that of single jobholders. Since a large fraction of multiple jobholders work one full- and one part-time job, this suggests that part-time wages are in general lower than full-time wages. Note the exception for workers with advanced degrees. For these workers a second job increases their hourly earnings.

Paxson and Sicherman (1996, Table 5) report the means and medians of the ratio of the second to main job wage rate for a variety of occupational groups. The evidence is, in the authors' words "sketchy" as to the magnitude of this ratio. For some groups the ratio is below one, indicating that the wage on the second job is lower than on the main job (e.g. the median ratio is 0.863 for "Operatives"). For some groups the ratio is above one, indicating that the wage on the second job is higher than on the main job (e.g. the median ratio is 1.293 for "college teachers"). The overall median ratio is 1.050.

The Paxson and Sicherman (1996) evidence is consistent with our results along two dimensions. First, the lack of a clear-cut ordering of main- and second-job wage rates is consistent with the heights of bars in Figure 4: for some workers the second job reduces their hourly earnings and for some workers it increases it. Second, they find that the median ratio of wages between second and main job is positively correlated with the skill level across a two-digit occupational classification. For example, the ratio is higher for university and college teachers compared to primary and secondary teachers. Likewise doctors, judges, lawyers, accountants, and self-employed managers tend to exhibit higher wage ratios. Workers who are neither professionals nor managers tend to have lower wage ratios.

3 Model

Time lasts for one period. There is a mass 1 of workers differentiated by their taste for leisure, $\alpha \in \mathbb{R}_+$, their efficiency units of labor, $z \in \mathbb{R}_+$ and their education $x \in \mathbb{X}$. The tuple (α, z, x) is a permanent feature for each worker.

There are two levels of education which we label "skilled" and "unskilled:" $\mathbb{X} = \{S, U\}$. There is a mass μ of skilled workers. Let G denote the distribution of α. We assume this distribution to be the same for skilled and unskilled workers. Let $Z(z|x)$ denote the distribution of efficiency units of labor conditional on education x. We assume that G and $Z(z|x)$ are independent.

A representative firm produces output via a constant-returns-to-scale technology using the services of skilled and unskilled labor allocated between full- and part-time jobs. A full-time job requires a fraction n_F of a worker's time. A part-time job requires a fraction n_P of time.

There are five employment types among which workers can choose:

$$e \in \mathbb{E} = \{F, P, FP, PP, FF\},$$

where F means "one full-time job," P means "one part-time job," FP means "one full-time job and one part-time job," etc. We restrict the number of jobs to either one or two since, as noted in Section 2, less than 1 percent of all multiple jobholders have more than two jobs.

3.1 Production

The firm's technology is given by

$$Y = F\left(L(S), L(U)\right), \tag{2}$$

e	$y_e(x)$	ℓ_e
F	$w_F(x)n_F$	$1-n_F$
P	$w_P(x)n_P$	$1-n_P$
FP	$w_F(x)n_F+w_P(x)n_P$	$1-n_F-n_P$
PP	$2w_P(x)n_P$	$1-2n_P$
FF	$2w_F(x)n_F$	$1-2n_F$

Table 2: Income per efficiency units of labor and leisure for worker with skill x by employment type

where $L(S)$ (and $L(U)$) aggregates labor from full- and part-time skilled (and unskilled) workers:

$$L(x)=\left(A_F(x)L_F(x)^{\phi(x)}+A_P(x)L_P(x)^{\phi(x)}\right)^{1/\phi(x)},\ x\in\{S,U\}. \tag{3}$$

The parameters $A_F(x)$ and $A_P(x)$ are skill- and job-specific technology parameters, and $\phi(x)\leq 1$ controls the elasticity of substitution between full-time and part-time labor. The terms $L_F(x)$ and $L_P(x)$ denote total labor inputs from full-time and part-time workers with skill x, respectively. The firm's optimization problem is

$$\max_{\{L_F(x),L_P(x)\}} Y-\sum_{x\in\mathbb{X}}w_F(x)L_F(x)-\sum_{x\in\mathbb{X}}w_P(x)L_P(x). \tag{4}$$

where $w_F(x)$ and $w_P(x)$ are wages per efficiency unit of labor. The firm's first-order conditions are

$$F_1(L(S),L(U))\frac{\partial L(S)}{\partial L_j(S)}=w_j(S),\quad \text{for } j\in\{F,P\}$$

and

$$F_2(L(S),L(U))\frac{\partial L(U)}{\partial L_j(U)}=w_j(U),\quad \text{for } j\in\{F,P\}.$$

3.2 Workers

Preferences are defined over consumption and leisure. A typical worker's preferences are represented by the utility function

$$U(c)+\alpha V(\ell),$$

where c and ℓ stand for consumption and leisure, respectively. The functions U and V are increasing, twice-continuously differentiable and concave utility indexes. Let $y_e(x)$ denote income per efficiency units of labor for a worker with education x and employment type e. A worker's labor income is $zy_e(x)$. Let ℓ_e denote leisure. Table 2 shows income and leisure for all $e\in\mathbb{E}$. The value function for worker (α,z,x) in employment type e is

$$W_e(\alpha,z,x)=U\left(zy_e(x)\right)+\alpha V\left(\ell_e\right). \tag{5}$$

A worker chooses the employment type yielding the highest utility, hence his labor supply is determined by

$$\max_{e \in \mathbb{E}} \quad W_e(\alpha, z, x). \tag{6}$$

It is useful to define $I_e(x)$, the set of workers with education x choosing employment type e:

$$I_e(x) = \{(\alpha, z) \in \mathbb{R}_+ \times \mathbb{R}_+ : W_e(\alpha, z, x) \geq W_{e'}(\alpha, z, x) \quad \forall e' \neq e\}.$$

The efficiency units of labor per hours worked, supplied on *one* job by workers with education x in employment type e is then

$$H_e(x) = \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} \mathbb{I}\{(\alpha, z) \in I_e(x)\} z dG(\alpha) dZ(z|x).$$

3.3 Labor market equilibrium

An equilibrium is a set of prices $\{w_j(x)\}$ for $(x, j) \in \mathbb{X} \times \{F, P\}$; an allocation for the firm $\{L_j(x)\}$ for $(x, j) \in \mathbb{X} \times \{F, P\}$; and an allocation of workers to jobs $\{I_e(x)\}$ for $(x, e) \in \mathbb{X} \times \mathbb{E}$ such that

1. The firm's allocation solves optimization problem (4) given prices;
2. The worker's allocation solves optimization problem (6) given prices;
3. Prices clear markets:

$$\begin{aligned}
L_F(S) &= \mu \left[H_F(S) + H_{FP}(S) + 2H_{FF}(S)\right] n_F, \\
L_P(S) &= \mu \left[H_P(S) + H_{FP}(S) + 2H_{PP}(S)\right] n_P, \\
L_F(U) &= (1 - \mu) \left[H_F(U) + H_{FP}(U) + 2H_{FF}(U)\right] n_F, \\
L_P(U) &= (1 - \mu) \left[H_P(U) + H_{FP}(U) + 2H_{PP}(U)\right] n_P.
\end{aligned}$$

3.4 Analysis

In this section we analyze labor supply, that is the type of employment workers choose. We discuss the conditions under which the model can simultaneously account for facts (i)-(iii).

We illustrate our discussion with a simplified version of a worker's decision problem: the choice between a full-time job ($e = F$) and a full- and a part-time job ($e = FP$). This simplifies the discussion while demonstrating the key mechanisms of the model. With two employment types there are two value functions to compare for each education group. With the five employment types we consider in the quantitative application of Section 4, there are five value functions to compare for each education group.

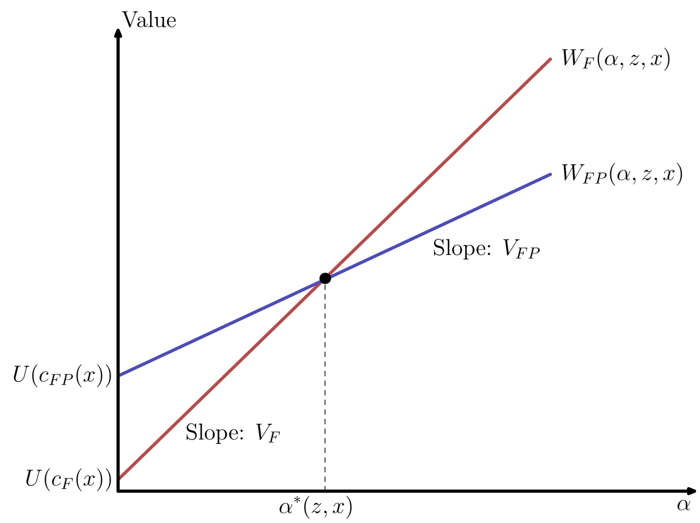


Figure 5: The determination of $\alpha^*(z, x)$

The value functions for workers in employment types F and FP are

$$W_F(\alpha, z, x) = U(zw_F(x)n_F) + \alpha V_F, \tag{7}$$

$$W_{FP}(\alpha, z, x) = U(z(w_F(x)n_F + w_P(x)n_P)) + \alpha V_{FP}. \tag{8}$$

where $V_F \equiv V(1 - n_F)$ and $V_{FP} \equiv V(1 - n_F - n_P)$. We define $\alpha^*(z, x)$ as the threshold preference such that a worker with ability z and education x is indifferent between F and FP:

$$W_{FP}(\alpha^*(z, x), z, x) = W_F(\alpha^*(z, x), z, x).$$

It follows that

$$\alpha^*(z, x) = \frac{U(c_F(x)) - U(c_{FP}(x))}{V_{FP} - V_F} \tag{9}$$

where $c_F(x) = zw_F(x)n_F$ and $c_{FP}(x) = z(w_F(x)n_F + w_P(x)n_P)$. It is convenient to write $c_{FP}(x) = c_F(x)\omega(x)$ where $\omega(x)$ is defined by

$$\omega(x) = 1 + \rho(x)n_P/n_F, \tag{10}$$

$$\rho(x) = w_P(x)/w_F(x). \tag{11}$$

Note that $c_{FP}(x) > c_F(x)$ and $V_{FP} < V_F$. It follows that $\alpha^*(z, x) > 0$. Figure 5 represents the determination of $\alpha^*(z, x)$: workers with ability z, education x and preferences $\alpha > \alpha^*(z, x)$ choose F.

The properties of $\alpha^*(z, x)$ determine labor supply and how it changes with productivity. We show below that these properties depend on which of the income or substitution effects dominate when productivity changes. We further claim that whether the income or substitution effect dominates depends on a property of the utility index U: if $U'(c\omega)\omega$ is decreasing in ω, then the income effect

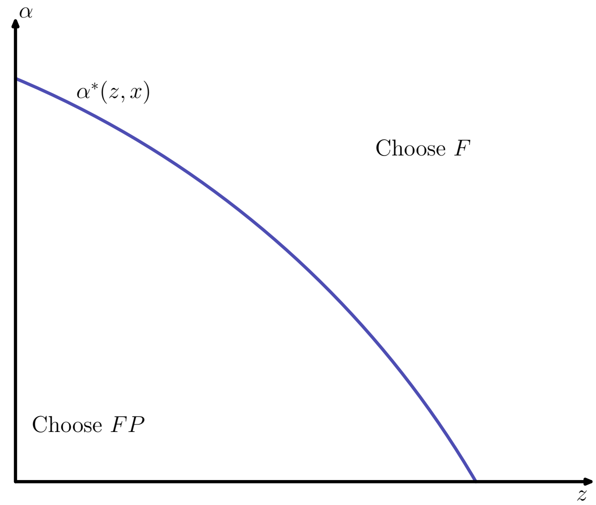


Figure 6: The employment choice when the income effect dominates

dominates. We prove this claim in Appendix A.

Equation (9) implies

$$\frac{d\alpha^*(z,x)}{dz} = \frac{w_F(x)n_F}{V_{FP} - V_F}\left[U'\left(c_F(x)\right) - U'\left(c_F(x)\omega(x)\right)\omega(x)\right]. \tag{12}$$

The following proposition establishes that this expression is negative if and only if preferences are such that the income effect dominates

Proposition 1. *$d\alpha^*(z,x)/dz < 0$ if and only if $U'(c\omega)\omega$ is decreasing in ω.*

The proof is immediate since $\omega(x) > 1$ and $V_{FP} < V_F$. The $\alpha^*(z,x)$-locus is described in Figure 6. Recall that the distribution of α is independent of the distribution of z. Figure 6 indicates, therefore, that the proportion of workers with two jobs decreases with z. That is, conditional on education, the most productive workers are the least likely to hold multiple jobs. This follows from the dominating income effect: More productive workers seek to work fewer hours (if the substitution effects dominated they would seek to work more hours) which, in our model, they can achieve at the extensive margin by working fewer jobs.

This property of our model when the income effect dominates explains fact (ii): the observed negative correlation between hourly wages and the prevalence of multiple jobholders conditional on education. To understand this, note that the hourly wage of a worker with employment type $e = F$ is $zw_F(x)$, while the hourly wage of a worker with $e = FP$ is

$$\frac{z(w_F(x)n_F + w_P(x)n_P)}{n_F + n_P} = zw_F(x)\frac{1 + \rho(x)\frac{n_P}{n_F}}{1 + \frac{n_P}{n_F}}.$$

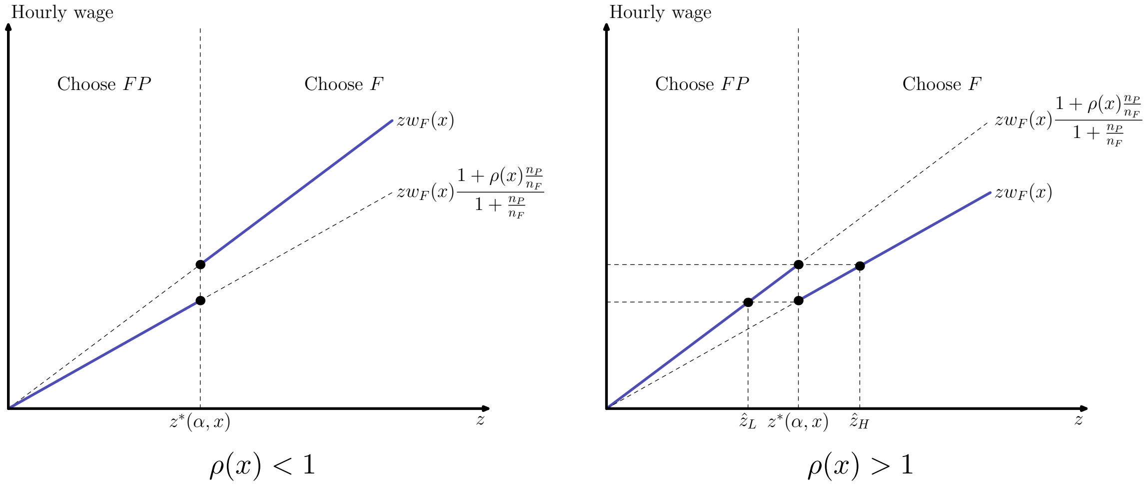


Figure 7: Hourly wage per employment type

Both hourly wages are increasing in z. Let $z^*(\alpha, x)$ denote the inverse (with respect to z) of $\alpha^*(z, x)$: for any α, workers with ability $z < z^*(\alpha, x)$ choose to work two jobs. Figure 7 shows that when $\rho(x) < 1$, workers with one job have higher hourly wages than workers with two jobs. When $\rho(x) > 1$ there is an ability range, $[\hat{z}_L, \hat{z}_H]$, inside which some workers with two jobs have higher hourly earnings than some with one job. This would not imply a positive correlation between the propensity to moonlight and hourly wage, however, since multiple jobholders with $z < \hat{z}_L$ have lower hourly wages than single jobholders with $z > \hat{z}_H$. In our calibration, we find $\rho(x) < 1$.

3.4.1 *The effect of productivity*

How do employment decisions change when $w_F(x)$ and/or $w_P(x)$ increase? Several effects must be considered. First, suppose the ratio $\rho(x) = w_P(x)/w_F(x)$ remains constant as both wages increase proportionally. Then, there are standard income and substitution effects at work. If the income effect dominates, workers tend to choose employment types requiring fewer hours of work. In the simplified model presented here this means one full-time job. Therefore, workers are less likely to work two jobs as the wages increase. If the substitution effect dominates the opposite occurs.

In addition to the standard income and substitution effects there can also be "relative price effects" when the ratio $\rho(x)$ changes. We use the term "relative price effect" to refer to the relative wages between full-time and part-time jobs, not to the relative price of leisure and consumption, which changes even when $w_F(x)$ and $w_P(x)$ increase proportionally. Suppose, for instance, that $w_F(x)$ is multiplied by a factor 2 and $w_P(x)$ is multiplied by a factor 3. This could be viewed first as multiplying both wages by a factor 2 and, second, as increasing $w_P(x)$ alone. The first part implies income and substitution effects as described earlier. In the second part, that is when $w_P(x)$ alone increases there is (i) a standard income effect because the worker becomes richer; (ii) a standard substitution effect because leisure becomes more expensive; and (iii) what we refer to as the "relative price effect" because time spent working the full time job is becoming relatively more expensive. The latter effect makes part-time labor more attractive and induces workers to choose to work part-time. In the simplified model presented here this means workers are more likely to work two jobs

In the general model with all five employment arrangements, choosing two jobs when $w_F(x)/w_P(x)$ decreases may or may not be optimal. It depends on the relative strength of the income and substitution effects. The "relative price effects" imply that when $w_F(x)/w_P(x)$ decreases workers tend to favor the part-time job relative to the full-time job. If the standard income effect is strong enough, workers would prefer to hold only one, part-time job. If, instead, the standard substitution effect is strong enough, workers would prefer to hold two part-time jobs. It follows that the interplay between income, substitution and relative price effects is ultimately a quantitative question that we address in Section 4. For now, we show formally how these effects operate in our simplified model.

The following proposition establishes that when the income effect dominates $\alpha^*(z,x)$ is decreasing in $w_F(x)$

Proposition 2. *$d\alpha^*(z,x)/dw_F(x)|_{\rho(x)\ constant} < 0$ if and only if $U'(c\omega)\omega$ is decreasing in ω.*

The proof follows from the expression

$$\left.\frac{d\alpha^*(z,x)}{dw_F(x)}\right|_{\rho(x)\text{ constant}} = \frac{zn_F}{V_{FP}-V_F}\left[U'\left(c_F(x)\right) - U'\left(c_F(x)\omega(x)\right)\omega(x)\right]$$

and the fact that $\omega(x) > 1$. Thus, a proportional increase in $w_F(x)$ and $w_P(x)$ lowers the $\alpha^*(z,x)$-locus and implies that fewer workers choose to hold two jobs (see Figure 6). This property of our model when the income effect dominates provides an explanation for fact (iii): the declining trend in multiple jobholdings in the U.S. data.

The following proposition establishes that $\alpha^*(z,x)$ is increasing in $\rho_(x)$.

Proposition 3. *$d\alpha^*(z,x)/d\rho(x)|_{w_F(x)\ constant} > 0$.*

The proof follows from the expression

$$\left.\frac{d\alpha^*(z,x)}{d\rho(x)}\right|_{w_F(x)\text{ constant}} = -\frac{1}{V_{FP}-V_F}U'\left(c_{FP}\right)zw_Fn_P.$$

Thus, an increase in $\rho(x)$ (holding w_F constant) raises the $\alpha^*(z)$-locus and implies that more workers choose to hold two jobs. (see Figure 6).

3.4.2 *The comparative advantage*

In this section we discuss how a comparative advantage of skilled workers in part-time jobs is necessary for the model explain fact (i): the higher proportion of multiple jobholders among skilled workers.

The proportion of workers with skill x and two jobs is

$$\mathrm{mjh}(x) = \int G(\alpha^*(z,x))dZ(z|x).$$

We proceed in two steps. First, we discuss the conditions under which $\alpha^*(z,S) > \alpha^*(z,U)$ is necessary for $\mathrm{mjh}(S) > \mathrm{mjh}(U)$. Second, we discuss the conditions under which a comparative advantage of skilled workers in part-time jobs, $\rho(S) > \rho(U)$, is necessary and sufficient for $\alpha^*(z,S) > \alpha^*(z,U)$.

1. Suppose that $Z(z|S) = Z(z|U) - \delta(z)$. If $\delta(z) = 0$ the distributions are identical. If $\delta(z) > 0$ then $Z(z|S)$ first-order stochastically dominates $Z(z|U)$. If $\delta(z) < 0$ the opposite prevails. Then,

$$\mathrm{mjh}(S) = \int G(\alpha^*(z,S))\,dZ(z|U) - \underbrace{\int G(\alpha^*(z,S))\,d\delta(z)}_{\Gamma}.$$

   Suppose that $\alpha^*(z,S) > \alpha^*(z,U)$, then

$$\int G(\alpha^*(z,S))\,dZ(z|U) = \int G(\alpha^*(z,U))\,dZ(z|U) + \Delta$$

   where $\Delta > 0$. Thus,

$$\mathrm{mjh}(S) - \mathrm{mjh}(U) = \Delta - \Gamma.$$

   This expression decomposes the skilled-unskilled difference in the proportion of multiple jobholders into skilled-unskilled differences in decisions, Δ, and skilled-unskilled differences in distributions, Γ. If $\delta(z) = 0$ and thus $\Gamma = 0$, then $\alpha^*(z,S) > \alpha^*(z,U)$ if and only if $\mathrm{mjh}(S) > \mathrm{mjh}(U)$. If $\delta(z) > 0$ and thus $\Gamma > 0$, then $\Delta > \Gamma$ if and only if $\mathrm{mjh}(S) > \mathrm{mjh}(U)$. That is, $\alpha^*(z,S)$ must be large enough relative to $\alpha^*(z,U)$. In sum, $\alpha^*(z,S) > \alpha^*(z,U)$ is necessary (not sufficient when $\Gamma > 0$) for $\mathrm{mjh}(S) - \mathrm{mjh}(U) > 0$.

2. Define (with some abuse of notations) $\alpha^*(z, w_F(x), \rho(x)) \equiv \alpha^*(z,x)$ then

$$\begin{aligned}\alpha^*(z,S) - \alpha^*(z,U) = &\overbrace{\alpha^*(z, w_F(S), \rho(S)) - \alpha^*(z, w_F(U), \rho(S))}^{A} \\ &+ \alpha^*(z, w_F(U), \rho(S)) - \alpha^*(z, w_F(U), \rho(U))\end{aligned}$$

   If there is a skill premium, i.e. if $w_F(S) > w_F(U)$, Proposition 2 implies $A < 0$. Then Proposition 3 implies that $\alpha^*(z,S) > \alpha^*(z,U)$ if and only if $\rho(S) > \rho(U)$, that is if skilled workers have a comparative advantage in part-time jobs.

Note that the only case in which the arguments developed above does not apply is when $Z(z|U)$ first-order stochastically dominates $Z(z|S)$, then $g(z) < 0$. This case is not relevant, however, since it implies that the average unskilled workers would be more skilled than the average skilled worker: $\int z dZ(z|U) > \int z dZ(z|S)$.

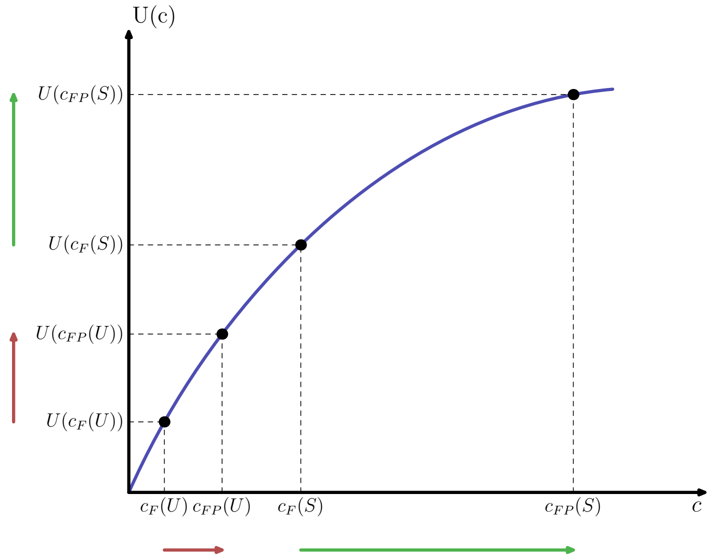


Figure 8: The comparative advantage

The economics behind this discussion are illustrated in Figure 8. Consider two workers, one skilled and the other unskilled. Suppose they both work one full-time job and have the same preference for leisure, α. Suppose also that the ability, z, of the skilled worker is no less than that of the unskilled worker. Given the skill premium and the difference in ability, $c_F(S) > c_F(U)$. Under what conditions would the skilled worker take a second job while the unskilled worker would not? The cost of taking the second job, forgone leisure time, is the same for each worker. The benefit, however, is not the same. The skilled worker's marginal utility is lower because of the skill premium and the possible difference in ability. Thus, in order for the utility gain from the second job to be larger for the skilled worker (green vertical arrow) than for the unskilled (red vertical arrow), the associated consumption gain must be larger for the skilled worker relative to the unskilled. Hence the need for a comparative advantage of skilled workers in part time jobs.

4 Quantitative analysis

4.1 Calibration

The model is calibrated to the data in 1994—the first year for which the CPS reports statistics on multiple jobholders. The calibrated moments are: the proportion of multiple jobholders by education, the proportion of workers with both a full- and a part-time job, the college premium, the variance of log-earnings conditional on education, the relative earnings of one- and two-job holders by education, and the coefficient on real wages in Probit models 2A and 2B. This implies 10 calibrated parameters to match 12 moments.

The time-period is set to one week, and we assume a total of $7 \times (24-8) = 112$ hours available for either work or leisure. A full-time job requiring 40 hours of work implies $n_F = 40/112 = 0.36$. Using the data presented in Figure 2, a part-time job requires 10 hours of work; therefore $n_P = 10/112 = 0.09$. In 1994, 56 percent of workers had at least some college education (CPS). Thus, we set $\mu = 0.56$.

For functional forms, the utility indexes are given by,

$$U(c) = \frac{c^{1-\sigma_C}}{1-\sigma_C} \text{ and } V(\ell) = \frac{\ell^{1-\sigma_L}}{1-\sigma_L},$$

where $\sigma_C, \sigma_L > 0$, and the production function is assumed to be

$$Y = \left(L(S)^{\eta} + L(U)^{\eta}\right)^{1/\eta}. \tag{13}$$

The elasticity of substitution between skilled and unskilled labor is $1/(1-\eta)$. We follow Goldin and Katz (2007) and use an elasticity of substitution of 1.6, implying $\eta = 1 - 1/1.6$. We use the same value for $\phi(S)$ and $\phi(U)$, which determine the elasticity of substitution between full-time and part-time labor for skilled and unskilled labor, respectively (Equation 3). While ideally $\phi(S)$ and $\phi(U)$ would be calibrated using data on the elasticity of substitution between full and part-time workers, no such data exists to the best of our knowledge. Note that we do not assign weights to $L(S)^{\eta}$ nor to $L(U)^{\eta}$ in Equation (13). Such weights could not be distinguished from the productivity parameters $A_j(x)$ for $(x, j) \in \mathbb{X} \times \{F, P\}$.

We assume $G(\alpha)$ is log-normal with mean μ_α and variance σ_α^2. Similarly, the ability distributions, $Z(z|x)$, are log-normal with mean $\mu_z(x)$ and variances $\sigma_z^2(x)$. We assume mean abilities are the same across education levels: $\mu_z(S) = \mu_z(U) = 1.0$. This is a normalization since differences in mean abilities cannot be distinguished from differences in skill-specific technological parameters, $A_j(x)$.

The 10 parameters to calibrate are:

$$\omega = \left\{\sigma_C, \sigma_L, \mu_\alpha, \sigma_\alpha^2, \sigma_z^2(S), \sigma_z^2(U), A_F(S), A_P(S), A_F(U), A_P(U)\right\}.$$

These 10 parameters are calibrated to 12 moments from the data. Specifically, we define

$$\Theta(\omega)=\begin{bmatrix}\sum_{e\in\text{mjh}} M_e(S)/0.0722\\ \sum_{e\in\text{mjh}} M_e(U)/0.0412\\ M_{FP}(S)/0.0380\\ M_{FP}(U)/0.0210\\ e_2(S)/e_1(S)/0.93\\ e_1(U)/e_1(S)/0.67\\ e_2(U)/e_1(S)/0.67\\ E(S)/E(U)/1.52\\ V(S)/0.62\\ V(U)/0.60\\ -\lambda(S)/0.09\\ -\lambda(U)/0.06\end{bmatrix} \tag{14}$$

where mjh $\equiv \{FF, FP, PP\}$, and find ω as the solution to

$$\min_{\omega}\left[\Theta'(\omega)-1\right]\left[\Theta(\omega)-1\right].$$

The model-implied moments in $\Theta(\omega)$ are formally described in Appendix B. The first two rows of $\Theta(\omega)$ indicate the distance between the model's implied proportions of skilled and unskilled workers with two jobs, and their empirical counterpart in 1994. The third and fourth rows are the model-to-data distances in the proportion of workers (skilled and unskilled) with a full- and a part-time jobs.[7] Rows 5-7 are the model-to-data distances between the hourly earnings of workers with 1 and 2 jobs by education. That is, $e_1(x)$ is the average hourly earnings of workers with education x working one job; $e_2(x)$ is for those working two jobs. Row 8 is the model-to-data distance of the college earnings premium. Rows 9 and 10 are the model-to-data distance of the variance of log earnings for skilled and unskilled workers. Finally, rows 11 and 12 are model-to-data distance of the slope of earnings per hour in a Probit regression where the dependent variable is whether a worker has one or two jobs—see Models 2A and 2B in Table 1.

Although the parameters are determined simultaneously, some moments matter more than others for certain parameters. The existence and strength of a dominating income effect depends on the utility indexes U and V (see Equation 12). Thus, the negative slope $\lambda(S)$ and $\lambda(U)$ have a first-order effect on the curvature parameters σ_C and σ_L. The percentage of workers working two jobs, and the percentage of workers with both a full- and a part-time job have first-order effects on the technology parameters, $A_j(x)$ who, in turn, determine the comparative advantage. The college premium also plays a role in the determination of the comparative advantage: the higher the college premium, the higher the comparative advantage needed to induce skilled worker to take on a second job. Finally, the variance of the distributions $G(\alpha)$ and $Z(z|x)$ have first-order effects on the dispersion of earnings.

[7] When calculating these proportions, we define a part-time job as fewer than 35 hours per week.

Table 3: Calibrated parameters

Preferences	$\sigma_C = 1.18$, $\sigma_L = 1.97$
	$\mu_\alpha = 1.64$, $\sigma_\alpha^2 = 0.35$
Abilities	$\mu_z(S) = 1.00$, $\sigma_z^2(S) = 0.91$
	$\mu_z(U) = 1.00$, $\sigma_z^2(U) = 0.85$
Technology	$\phi(S) = 0.38$, $\phi(U) = 0.38$, $\eta = 0.38$
	$A_F(S) = 1.11$, $A_F(U) = 0.73$
	$A_P(S) = 0.13$, $A_P(U) = 0.04$
Worktime	$n_F = 0.36$, $n_P = 0.09$

Table 4: Model fit

Moment description		Data	Model
%age multiple jobholders, S	$\sum_{e \in \text{mjh}} M_e(S)$	7.22%	7.46%
%age multiple jobholders, U	$\sum_{e \in \text{mjh}} M_e(U)$	4.12%	3.58%
%age FP, S	$M_{FP}(S)$	3.80%	3.43%
%age FP, U	$M_{FP}(U)$	2.10%	2.40%
Relative earnings	$e_2(S)/e_1(S)$	0.93	1.07
Relative earnings	$e_1(U)/e_1(S)$	0.67	0.79
Relative earnings	$e_2(U)/e_1(S)$	0.67	0.61
Col. premium	$E(S)/E(U)$	1.52	1.40
Var. log earnings, S	$V(S)$	0.62	0.61
Var. log earnings, U	$V(U)$	0.60	0.59
Probit slope, S	$\lambda(S)$	-0.09	-0.09
Probit slope, U	$\lambda(U)$	-0.06	-0.06

Table 3 gives the calibrated moments. and Table 4 indicates the model fit. The calibrated model reproduces the key motivating features of moonlighting and income differences in the data. First, conditional on education the probability that a worker holds two jobs is decreasing in hourly earnings: both $\lambda(S)$ and $\lambda(U)$ are negative. Despite this negative correlation, however, college-educated workers are more likely to hold two jobs than high school-educated worker even though there is a college premium.

4.2 Discussion

Figure 9 displays aggregate labor supply measured in hours for skilled and unskilled workers. We make several observations. First, the black dot represents the calibrated equilibrium. Second, along the red lines the ratio $\rho(x)$ is constant at its equilibrium value. Thus, as workers face increasing wages and a constant relative price of full-time versus part-time jobs, total hours worked decreases. This is

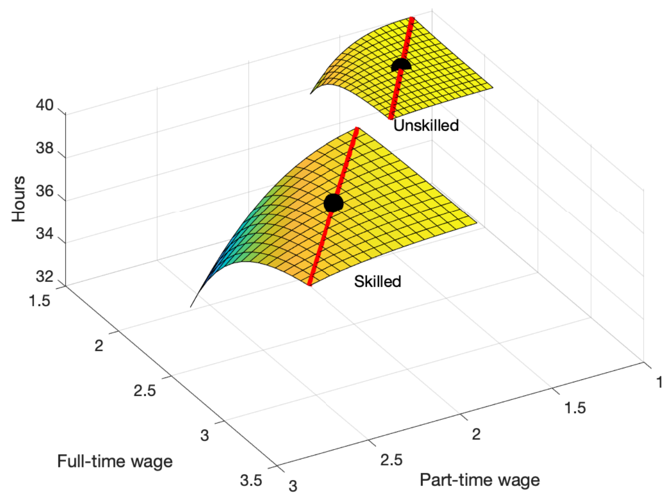


Figure 9: Labor supply

because the income effect from an increase in wages dominates and workers seek to reduce their hours by selecting into employment types requiring fewer hours. Second, note that labor supply is higher for unskilled workers than for skilled workers, while skilled workers are more likely to hold multiple jobs. Again, this results from the income effect: unskilled workers seek to work longer hours because they are paid less on average. They achieve this by selecting into employment types that require long hours: one full-time job, one full-time and one part-time job or two full-time jobs. Skilled workers, on the other hand, seek to work fewer hours, but are enticed to choose multiple jobs by their comparative advantage in part-time jobs. Thus, two part-time jobs are more prevalent among skilled workers. Since hours on two part-time jobs do not add up to the hours of one full-time job, this tends to lower the total hours supplied by skilled workers.

Before turning to the time series implications of the model, we examine how sensitive multiple jobholding is with respect to the exogenous parameters. Understanding these sensitivities is enlightening because, in the time-series experiments we allow the exogenous variables to changes in ways consistent with data. As a result, it is difficult to assess whether a variable has a "small" effect because the elasticity of the model with respect to this variable is "small," or because the change in the variable is "small" in the data. To accomplish this, we consider a 1 percent change relative to calibrated value for each exogenous variable. Namely, we multiply each exogenous variable by 1.01, leaving the other variables constant. We then compute the relative (in percent) change in the proportion of multiple jobholders for each education group. Table 5 shows the results.

A few observations about Table 5 are worth making. First, an increase in full-time productivity $A_F(x)$ reduces the proportion of multiple jobholders for both education groups, with the strongest effect being on group x. This is because an increase in $A_F(x)$ raises all wages and, thus, fewer workers seek to work two jobs.

Table 5: Elasticities

Variable	Skilled	Unskilled
$A_F(S)$	-2.20	-0.87
$A_P(S)$	1.17	-0.01
$A_F(U)$	-0.32	-2.56
$A_P(U)$	-0.00	1.17
μ	0.28	-0.79

Note: The table reports the percentage change in the proportion of multiple jobholders after a one percent increase in one exogenous variable at a time.
Source: Authors' calculations.

Second an increase in part-time productivity $A_P(x)$ raises the proportion of multiple jobholders with education x and lower that proportion for workers in the opposite education group. As with $A_F(x)$, an increase $A_P(x)$ raises all wages. This could lead to a reduction in the proportion of multiple jobholders but it also makes part-time jobs relatively more attractive to workers with education x and, since very few workers work one part-time job ($e = P$), it follows that the proportion of multiple jobholders with education x increases.

Third, an increase in the proportion of skilled workers, μ, increases the proportion of skilled multiple jobholders and reduces the proportion of unskilled multiple jobholders. This obtains because, all else equal, when there are more skilled workers and fewer unskilled workers, wages decrease for the skilled and increase for the unskilled. The income effect then induces the skilled to work longer hours (and therefore to work more jobs) and the unskilled to work fewer hours (and therefore to work fewer jobs).

4.3 Time series implication

We compute a final equilibrium corresponding to the U.S. in 2017 and compare it to the initial equilibrium (1994). Two sets of parameters change between the 1994 and the 2017 equilibria. First, the proportion of college-educated workers increases from 56 to 66 percent, thus $\mu = 0.66$ in the final equilibrium. Second, productivity changes. To discipline the change in productivity we impose that productivity growth is the same for workers of a given skill, regardless of whether their employment is full-time or part-time. Denoting the growth rate of productivity for education x as $g(x)$:

$$A_j(x)|_{\text{final}} = (1 + g(x))A_j(x)|_{\text{initial}} \text{ for } j \in \{F, P\}$$

We find $g(S)$ and $g(U)$ such that the final equilibrium satisfies two conditions. First BEA data reveals that the real Gross Domestic Product per capita in the U.S. was 42% higher in 2017 than in 1994; Second, CPS data reveal that the college premium increases by 7% between 1994 and 2017. These conditions imply $g(S) = 0.24$ and $g(U) = -0.11$. Thus, the productivity of skilled workers increases (by 0.94% per year) while that of unskilled workers decreases (by 0.50% per year). It is important

Table 6: Time series

	Initial equilibrium	Intermediate equilibrium	Final equilibrium
%age multiple jobholders, S	7.46	6.44 (57)	6.45 (56)
%age multiple jobholders, U	3.58	2.71 (81)	2.87 (67)

to note, however, that all wages increase between the initial and final steady state. This is because, despite lower productivity the marginal product of unskilled workers does not decline. There are two reasons for this: there are more skilled workers in the final equilibrium, and they are more productive. Both effects contribute to the increase in the marginal productivity of unskilled workers.

Table 6 presents the result of this experiment in two steps. The middle column ("Intermediate equilibrium") represents an equilibrium where the proportion of college-educated workers and the college premium are constant at their 1994 values, while the gross domestic product increases by 42%. Thus, we label the change from the initial to the intermediate equilibrium as the "economic growth effect," and the change from the intermediate to the final equilibrium as the "education effect."

The figures in parenthesis indicate the proportion of the observed change that is accounted for by the model. In the data, the proportion of skilled multiple jobholders decreased by 1.8pp (from 7.22 to 5.42%) between 1994 and 2017. The economic growth effect implies a decrease of 1.02pp (from 7.46 to 6.44%), accounting for 57% (1.02/1.8) of the observed change. The combined effects of economic growth education imply a decrease of 1.01pp (from 7.46 to 6.45%), accounting for 56% of the observed change. The observed decline in the proportion of multiple jobholders among unskilled workers is 1.06pp (from 4.12 to 3.06%). The contribution of economic growth alone accounts for 81% of the observed decline for these workers, while the combined effects of growth and education accounts for 67% of the observed decline.

5 Conclusion

We documented three facts: (i) higher educated workers are more likely to work multiple jobs; (ii) conditional on education, workers with higher wages are less likely to work multiple jobs; and (iii) the proportion of multiple job-holders is declining over time for all education groups. These facts present a challenge to explain: they indicate a positive correlation between moonlighting and income across education groups, while the correlation is negative within education groups. We developed a quantitative theory to explain facts (i), (ii) and (iii)

We show that a dominating income effect explains facts (ii) and (iii), but is inconsistent with fact (i). Fact (i) can be consistent only if skilled workers have a comparative advantage in part-time jobs, which causes them to be more likely to hold multiple jobs. Hence the combination of a dominating income effect and a comparative advantage explains fact (i), (ii) and (iii).

The model is calibrated to 1994 cross-sectional data. In particular, we estimate the slopes of the probability of moonlighting with respect to wages, conditional on education, in cross-sectional Probit models. Second we use these slopes, among other 1994 cross sectional moments, to calibrate our model. That is, we estimate the same Probit equations on model-generated data, and match the slopes to their empirical counterparts. We finally assessed the ability of the model to reproduce the 2017 data. The model accounts for 56% of the moonlighting trend for skilled workers, and 67% for unskilled workers.

References

Aguiar, Mark, Mark Bils, Kerwin Kofi Charles, and Erik Hurst, "Leisure luxuries and the labor supply of young men," *NBER Working Paper Series*, 2017, (23552).

Chang, Yongsung, Sun-Bin Kim, Kyooho Kwon, and Richard Rogerson, "Cross-Sectional and Aggregate Labor Supply," *ISER DP*, 2019, (1063).

Goldin, Claudia and Lawrence F. Katz, *The race between education and technology*, Belknap Press, Cambridge, MA, 2007.

Greenwood, Jeremy, Ananth Seshadri, and Mehmet Yorukoglu, "Engines of liberation," *The Review of Economic Studies*, 2005, *72* (1), 109–133.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L Violante, "Consumption and labor supply with partial insurance: An analytical framework," *American Economic Review*, 2014, *104* (7), 2075–2126.

Heckman, James J, "The common structure of statistical models of truncation, sample selection and limited dependent variables and a simple estimator for such models," in "Annals of Economic and Social Measurement, Volume 5, number 4," NBER, 1976, pp. 475–492.

Hirsch, Barry T and John V Winters, "Rotation group bias in measures of multiple job holding," *Economics Letters*, 2016, *147*, 160–163.

_ , Muhammad M Husain, and John V Winters, "Multiple job holding, local labor markets, and the business cycle," *IZA Journal of Labor Economics*, 2016, *5* (1), 4.

Kimmel, Jean and Karen Smith Conway, "Who moonlights and why? Evidence from the SIPP," *Industrial Relations: A Journal of Economy and Society*, 2001, *40* (1), 89–120.

Kopecky, Karen A, "The trend in retirement," *International Economic Review*, 2011, *52* (2), 287–316.

Lalé, Etienne, "The evolution of multiple jobholding in the US labor market: The complete picture of gross worker flows," November 2016. IZA Discussion Paper No. 10355.

_ , "Search and multiple jobholding," 2019. Manuscript.

McGrattan, Ellen R and Richard Rogerson, "Changes in hours worked, 1950-2000," *Federal Reserve Bank of Minneapolis Quarterly Review*, 2004, *28* (1), 14–33.

Ngai, L Rachel and Christopher A Pissarides, "Trends in hours and economic growth," *Review of Economic dynamics*, 2008, *11* (2), 239–256.

Paxson, Christina H and Nachum Sicherman, "The dynamics of dual job holding and job mobility," *Journal of labor economics*, 1996, *14* (3), 357–393.

Pijoan-Mas, Josep, "Precautionary savings or working longer hours?," *Review of Economic dynamics*, 2006, *9* (2), 326–352.

Restuccia, Diego and Guillaume Vandenbroucke, "Explaining educational attainment across countries and over time," *Review of Economic Dynamics*, 2014, *17* (4), 824–841.

Rogerson, Richard, "Structural transformation and the deterioration of European labor market outcomes," *Journal of political Economy*, 2008, *116* (2), 235–259.

Shishko, Robert and Bernard Rostker, "The economics of multiple job holding," *The American Economic Review*, 1976, *66* (3), 298–308.

Vandenbroucke, Guillaume, "Trends in Hours: The US from 1900 to 1950," *Journal of Economic Dynamics and Control*, 2009, *33* (1), 237–249.

A Income and substitution effects

In a standard (continuous choice) consumption-leisure tradeoff of the form $\max\{U(c)+V(\ell) : c = w(1-\ell)\}$, the solution must satisfy $U'(c)c = V'(\ell)(1-\ell)$. The right-hand side is decreasing in ℓ. Thus, when w increases and consumption increases as well, leisure increases if and only if $U'(c)c$ is decreasing. Thus, $U'(c)c$ decreasing implies that the income effect dominates. Note that (i) $\frac{d}{d\omega}U'(c\omega)\omega = U''(c\omega)c\omega + U'(c\omega)$; and (ii) $\frac{d}{dc}U'(c)c = U''(c)c + U'(c)$. It follows that

$$\frac{d}{dc}U'(c)c < 0 \Leftrightarrow \frac{d}{d\omega}U'(c\omega)\omega < 0.$$

B Model statistics

We describe formally the moments used in the calibration as follows. The mean and variances of log-earnings for workers with education x and employment type e are

$$\begin{aligned} E(x) &= \int_{\mathbb{R}_+}\int_{\mathbb{R}_+} \mathbb{I}\{(\alpha, z) \in I_e(x)\} \ln(zy_e(x)) dG(\alpha) dZ(z|x) \\ V(x) &= \int_{\mathbb{R}_+}\int_{\mathbb{R}_+} \mathbb{I}\{(\alpha, z) \in I_e(x)\} \left[\ln(zy_e(x)) - E(x)\right]^2 dG(\alpha) dZ(z|x). \end{aligned}$$

We let $M_e(x)$ denote the mass of workers with education x and employment type e:

$$M_e(x) = \int_{\mathbb{R}_+}\int_{\mathbb{R}_+} \mathbb{I}\{(\alpha, z) \in I_e(x)\} dG(\alpha) dZ(z|x). \tag{15}$$

The average hourly earnings for workers with education x and one job is then

$$e_1(x) = \frac{w_F(x)H_F(x) + w_P(x)H_P(x)}{M_F(x) + M_P(x)}.$$

Similarly, the average hourly earnings for workers with education x and two jobs is

$$e_{x,2} = \frac{\frac{w_F(x)n_F + w_P(x)n_P}{n_F + n_P} H_{FP}(x) + w_F(x)H_F(x) + w_P(x)H_P(x)}{M_{FP}(x) + M_{FF}(x) + M_{PP}(x)}.$$

The last two moments are computed from estimating the equivalent of model (1) on model-generated data.

C Data appendix

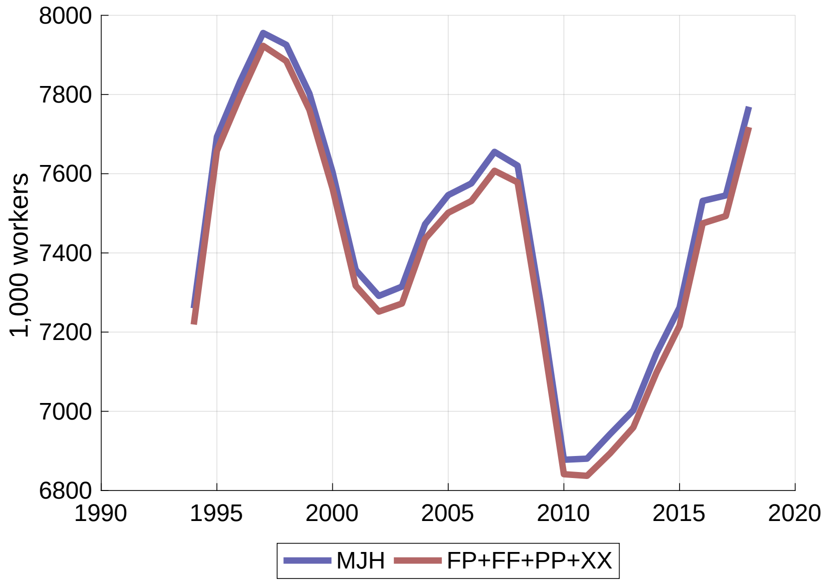


Figure C.1: Number of workers with two jobs

Note: MJH reports the number of multiple jobholders. The term FP+FF+PP+XX refer to the sum of people holding one full-time and one part-time job (FP), two full-time jobs (FF), two part-time jobs (PP), or two jobs with variable hours on either the primary or the secondary job (XX). The difference between the two lines indicates the number of workers with more than two jobs.
Source: Bureau of labor statistics.

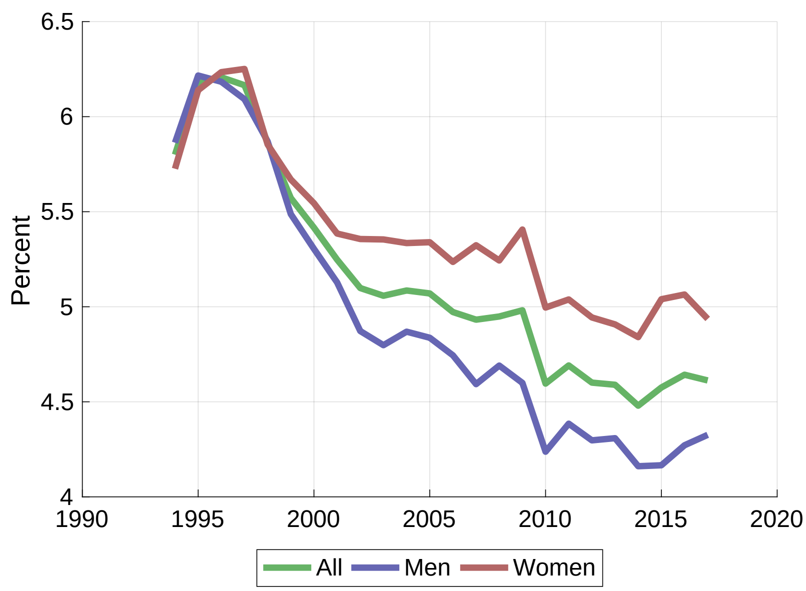


Figure C.2: Proportion of employees with multiple jobs

Source: Current Population Survey.

Table C.1: Probit Model Multiple Jobholding Dependent Variable

Variables	Model 3 PT Main	Model 3 FT Main
ln wage	0.0347	-0.0402
	(0.0294)	(0.0311)
Education-Log wage Interactions (Less than HS reference group)		
H.S.#Logwage	-0.0246	-0.0136
	(0.0332)	(0.0305)
Some College#Logwage	-0.0663*	-0.0331
	(0.0342)	(0.0294)
College#Logwage	-0.148***	-0.125***
	(0.0334)	(0.0301)
Advanced#Logwage	-0.0706**	-0.0584**
	(0.0359)	(0.0289)
Education (Less than HS reference group)		
HS	0.247***	0.229***
	(0.0833)	(0.0822)
Some college	0.542***	0.462***
	(0.0874)	(0.0806)
College	0.897***	0.769***
	(0.0864)	(0.0840)
Advanced	0.840***	0.650***
	(0.0953)	(0.0800)
Female	-0.128***	-0.129***
	(0.00965)	(0.00688)
Number of children	-0.00881***	0.0118***
	(0.00331)	(0.00269)
Married	-0.0923***	-0.121***
	(0.0105)	(0.00768)
Hrs. Vary	-0.201***	-0.141***
	(0.0116)	(0.0144)
Age (Older than 60 reference group)		
Age 20-29	0.150***	-0.00648
	(0.0147)	(0.0133)
Age 30-39	0.204***	0.0590***
	(0.0135)	(0.0112)
Age 40-49	0.291***	0.116***
	(0.0127)	(0.00811)
Age 50-59	0.258***	0.103***
	(0.0116)	(0.00816)
Race (White reference group)		
Black	-0.111***	0.000686
	(0.0192)	(0.0161)
Hispanic	-0.194***	-0.124***
	(0.0155)	(0.0154)
Other	-0.193***	-0.153***
	(0.0240)	(0.0230)
Constant	-1.663***	-1.570***
	(0.0785)	(0.0896)
Observations	472,793	1,523,734
State FE	Yes	Yes
Year FE	Yes	Yes

Standard errors (clustered at the state level) in parentheses. We define a part-time job as less than 35 hours per week.
*** p<0.01, ** p<0.05, * p<0.1

D Dynamic model with stochastic arrival and destruction of multiple jobs

The analyses in Lalé (2019) and Paxson and Sicherman (1996) suggest moonlighting to be a relatively short-lived phenomenon. While only around 5-7% of the employed population holds a second job at any point in time, closer to 50% of all employed individuals will hold one at some point in their working lives. This implies that second jobs do not last as long as primary jobs. In this appendix, we introduce a dynamic model where second jobs arrive and are destroyed stochastically. We make two points: First, the theoretical insights derived from the static model are preserved in the dynamic model, and the latter does not offer any further insights and/or mechanisms. Second, our analysis also shows that the dynamic model is not a better tool for the empirical analysis of moonlighting.

Time is discrete and lasts forever. Workers are infinitely lived. They have a common discount factor β and are differentiated by (α, z, x) as in Section 3. To illustrate the dynamic model's mechanisms we assume that all workers work one full-time job and receive a job offer for a second part-time job with probability $\delta(x)$. They may accept or decline the second job offer. Part-time jobs are destroyed with probability $\lambda(x)$. Upon destruction workers return to the full-time job only. Earnings and leisure on full-time and part-time jobs are exactly as in Section 3.

The value functions are

$$W_F(\alpha, z, x) = U(c_F(x)) + \alpha V_F + \beta \left[\delta(x) \max \left\{ W_{FP}(\alpha, z, x), W_F(\alpha, z, x) \right\} + (1 - \delta(x)) W_F(\alpha, z, x) \right], \tag{16}$$

$$W_{FP}(\alpha, z, x) = U(c_{FP}(x)) + \alpha V_{FP} + \beta \left[\lambda(x) W_F(\alpha, z, x) + (1 - \lambda(x)) W_{FP}(\alpha, z, x) \right]. \tag{17}$$

The marginal worker is defined by $W_{FP}(\alpha^*(z, x), z, x) = W_F(\alpha^*(z, x), z, x)$, implying

$$W_F(\alpha^*(z, x), z, x) = \frac{1}{1 - \beta} \Big[U(c_F(x)) + \alpha^*(z, x) V_F \Big] \tag{18}$$

$$W_{FP}(\alpha^*(z, x), z, x) = \frac{1}{1 - \beta} \Big[U(c_{FP}(x)) + \alpha^*(z, x) V_{FP} \Big] \tag{19}$$

Note that apart for the discounting term these equations are identical to Equations (7) and (8). It follows that the marginal worker is determined in the same way in the static and dynamic models:

$$\alpha^*(z, x) = \frac{U\left(c_F(x)\right) - U\left(c_{FP}(x)\right)}{V_{FP} - V_F} \tag{20}$$

which is identical to Equation (9). While the arrival and destruction rates of the second job potentially depend on skill level, they do not affect the decision to moonlight or not. This is intuitive, as either the additional consumption value of the second job outweighs the additional time, or it does not. If it does today, it will in the future. If $\delta(x)$ was endogenously determined by firm vacancy decisions, and wages determined by worker-firm interactions (e.g. competitive search, wage posting, or Nash Bargaining), workers would still take them as given and $\alpha^*(z, x)$ would still be determined by Equation (20).

It could be that the higher prevalence of multiple job-holding among skilled workers results from a higher δ and/or a lower λ. Lalé (2016), however, shows that the separation rates between education groups (college and no-college) are quite similar, leaving the different rates of multiple jobholding to differences in the willingness to work them, i.e., differences in $\alpha^*(z, x)$.

Equilibrium in Dynamic Model

To see the relationship between the static and dynamic models, consider the steady state equilibrium in the dynamic model. The labor market equilibrium, in each period, is defined similarly to that in the static model. That is, the labor market must clear in efficiency units of labor, for each skill type. Here we focus on characterizing the proportion of multiple jobholders by skill level. Towards this end, we normalize the population size to one, and denote the fraction of workers with skill level x by $\mu(x)$. Then, denote the density of workers, with efficiency units z and skill level x, in employment type $e \in \{F, FP\}$ by $N_e(z, x)$. It is useful to compare the density of workers across employment states in the dynamic model to our baseline static model in Equation (15). In the static model, $M_e(z, x)$ is determined by choices, $\mathbb{I}\{(\alpha, z) \in I_e(x)\}$, and the density over z. In the dynamic model, $\mathbb{I}\{(\alpha, z) \in I_e(x)\}$ still matters, but the density of workers in a particular employment state also depends on flows into and out of multiple jobs; therefore, $N_e(z, x)$ includes the population density, $G(z, x)$ (as described below). In particular, we have

$$M_e(x) = \int_{\mathbb{R}_+} N_e(z, x) dz$$

In the steady state equilibrium, for each z and x combination, the flows of workers between employment states $e = F$ and $e = FP$ must be equal. Thus,

$$\delta(x) G\big[\alpha^*(z, x)\big] N_F(z, x) = \lambda(x) N_{FP}(z, x) \tag{21}$$

$$N_F(z, x) + N_{FP}(z, x) = \frac{dZ(z|x)}{dz} \tag{22}$$

where $\dfrac{dZ(z|x)}{dz}$ denotes the pdf of the distribution $Z(z|x)$. Equation (21) equates the flows into and out of multiple jobs and Equation (22) ensures the total number of workers with (z, x) is consistent with the population distributions. Solving this for $N_{FP}(z, x)$ gives,

$$N_{FP}(z, x) = \frac{\frac{dZ(z|x)}{dz} \delta(x) G\big[\alpha^*(z, x)\big]}{\lambda(x) + \delta(x) G\big[\alpha^*(z, x)\big]} \tag{23}$$

In the static model, the proportion of workers with skill x and two jobs is given by $\text{mjh}(x) =$

$\int G(\alpha^*(z,x))dZ(z|x)$. In the dynamic model, the corresponding moment is given by,

$$\text{mjh}(x) = \int N_{FP}(z,x)dz = \int \left(\frac{\delta(x)G\big[\alpha^*(z,x)\big]}{\lambda(x) + \delta(x)G\big[\alpha^*(z,x)\big]} \right) dZ(z|x) \tag{24}$$

Examining Equation (24) from the dynamic model, differences in multiple jobholding by education and changes over time could result from differences in $\alpha^*(z,x)$ as in the static model, or now potentially on differences in $\delta(x)$ or $\lambda(x)$. The work of Lalé (2016) rules out a role for $\lambda(x)$, leaving either $\delta(x)$ or $G(\alpha^*(z,x))$ as vehicles to explain the differences/changes we highlight in the data. Since $\delta(x)$ and $G(\alpha^*(z,x))$ cannot be separately identified, adding the additional parameter in the dynamic model does not provide a better empirical model of moonlighting relative to the static model. This is not to say that quantifying the separate contributions of δ and G is not a worthwhile exercise, but simply that it remains well out of the scope of the questions taken up here.