Fertility Shocks and Equilibrium Marriage-Rate Dynamics

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Abstract

Female marriage probabilities were 50% higher in France in the years after World War 1, despite a large drop in the sex ratio. We develop a model of marital matching in which composition effects in the singles pool affect post-disruption matching rates. When calibrated to French data from World War 1, this mechanism explains 2/3 of the post-war rise in female marriage probabilities as the result of better composition of the pool of single men. We conclude that endogeneity issues make the sex ratio a potentially unreliable indicator of female marriage prospects.

JEL: D10, E13, J12, J13, and O11

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1 Introduction

Finding a marriage partner takes time; as in labor markets, the empirical observation that at any point in time parties on both sides of the market remain unmatched is often explained as the result of matching frictions. A central feature of frictional models of two-sided matching is that the effect of a change in the number of unmatched agents on the speed of matching is a function only of the ratio of the number of agents on each side of the market and the effect on each side has opposite sign. This feature, which arises from the assumption of constant returns to scale (CRS) in the matching process, implies that workers are quicker to find jobs when the vacancy unemployment ratio is high, and women quicker to marry when the "sex ratio," the ratio of men to women in the singles pool is high. Exogenous shocks to these ratios are rare however, so the hypothesis is not easy to verify.

The population of young single men (ages 20-40) in European countries declined abruptly as a result of World War 1; in France after the war, the sex ratio of singles in this age group was 28% lower than before the war. This decline constitutes an exogenous and unexpected shock to the marriage market; because France and Germany had universal male military conscription, the death rate of young males in these countries was essentially orthogonal to marital status. What makes France unique however is the availability of annual age-specific population data, by sex and marital status, starting from 1900. This combination of circumstances makes post-war France a rare opportunity to learn about the aggregate dynamics of the marriage-matching process. Previous papers, such as Vincent (1946), Henry (1966), and Abramitzky et al. (2011) have stressed the impact of the post-war sex ratio on lifetime marital outcomes. If the sex-ratio decline is indeed the main shock to the post-war marriage market, we should expect annual post-war marriage probabilities (the "marriage hazard rates") in France to be lower for women and higher for men than before the war.

In this paper, we show that the post-war marriage hazards do not fit the predicted pattern. We document the time path, by age and sex, of the marriage hazards in France from 1905 to 1930. We find that these increased dramatically for both sexes after the war, and that, contrary to the standard theory, a striking rise in the female marriage hazards accounts for a large share of the post-war rise in per-capita marriage rates. We show that this was more than a transitory blip in the data; five years after the war's end, female marriage hazards were still above the pre-war average, so the contradiction between theory and data cannot be dismissed as an aberration driven by conditions at the end of the war. We also find that the peak in female hazards cannot be explained away by the post-war formalization of war-time matches, as the response of unmarried births is implausibly small.

In principle, it would be possible to explain the rise in the post-war female hazards rates by assuming increasing returns to scale (IRTS) in the matching process. The higher-post-war prevalence of singles would then generate higher matching hazards for both sexes, eliminating the contradiction between theory and data. However an important drawback of IRTS would be the loss, due to multiple equilibria, of the predictive power of the CRS model. Furthermore, the preponderance of empirical work on matching, as documented by Pissarides and Petrongolo (2001) (for labor markets) and Botticini and Siow (2008) (for marriage markets) supports the CRS assumption. Finally, the fact that the peace-time marriage rate is very stable in France over the period from 1800-1950, despite several large war-time shocks, would be difficult to reconcile with the multiplicity of steady states implied by IRTS models.

Instead, we propose that a second war-related shock, the war-time "marriage bust," accounts for the post-war rise in the female marriage hazards. The per-capita marriage rate over the 1914-1918 period fell abruptly to an average 51% of the peace-time rate. Our idea is that this prolonged interruption of the matching process resulted in a radical change in composition of the pool of post-war singles. To simplify, suppose that singles differ in their marriage "propensity," which is public information. Normally, high-propensity singles would be comparatively rare in the singles pool, because they would exit into marriage relatively quickly. Now consider an extreme case: suppose that marriage rates fall to zero during war time. Then the high-propensity singles would tend to accumulate, and in the post-war singles pool would be unusually abundant, raising the average marriage hazard. In the presence of matching frictions, it could take considerable time for this composition effect to dissipate.

To formalize this mechanism in a more realistic context, our paper proposes a matching model in which heterogeneity in marriage propensity is derived from age-related differences in the utility from single life, and, among women, in the propensity to bear children. Following recent advances in the labor-matching literature, we use the competitive-search assumption to incorporate this heterogeneity into the matching process. We use French Census and Vital Statistics reports to calibrate the model's steady-state to the relevant age-specific statistics from pre-war France: the marriage hazards by sex, the sex ratio of singles, and the birth hazards of married women. We then model the war as a pair of unanticipated shocks of known duration, to male mortality and to fertility, which we calibrate so that the model matches the age distribution of military deaths and the war-time decline in marriage hazards. The empirical foundations of these shocks are well-established; the age pattern of military deaths is based on the tabulations of Vallin (1973), and the fertility shock reflects the war-time decline in per-capita birth rates based on the tabulations of Mitchell (1998), which Vandenbroucke (2014) shows is likely to have been a response to the risk of paternal mortality leading to single motherhood.

We then compute the equilibrium time path of the marriage market from the start of the war until the post-war return to the steady state. We find that the model generates a significant post-war increase in female marriage hazards, because the negative effect of the sex-ratio decline is offset by a larger, positive effect due to the war-time accumulation of single males who would normally have married much earlier. In fact our baseline calibration accounts for 66% of the post-war peak in female marriage hazards. The model also generates a leisurely transition back to the steady state, accounting for 38% of the half life of the post-war peak for women. Although the model peak occurs in 1919, a year before the peak in the data, the model successfully predicts the sex ratio of singles throughout the 1920s. Thus our baseline calibration succeeds in accounting for the basic features of the post-war transition dynamics in the marriage market.

Our results also suggest that the main force underlying the rise in the female marriage hazard is the post-war abundance, per single woman, of men with a high propensity to marry. There is no corresponding increase of high-propensity women, because the calibrated model implies that women's marriage propensity falls with age. Furthermore, we find that our results do not rely on minimizing the impact of military mortality: we show that the male-mortality shock, on its own, delivers large postwar *decline*s in the female marriage hazard[2].

Finally we ask whether the exceptional size of the female post-war peak might be due in part to pro-natalist sentiment in the post-war, as manifested by political events such as the post-war repudiation of feminist policies and the adoption of legislation favoring large families (see Roberts, 1994 and Robert, 2005). We show that births per married woman increased sharply relative to trend, a change that persisted well into the 1930s. When we augment the model with a post-war fertility shock, recalibrated to match the post-war birth-rate bump, the resulting transition path explains essentially all of the post-war peak in the female marriage hazards. Thus fertility plays key roles in generating variation in marriage propensities, both in cross-section and in time-series.

1.1 Related Literature

There is a long tradition in demography, dating from Groves and Ogburn (1928), of studying the causal effect of the sex ratio on marriage rates. In economics, the seminal work is the Becker (1973) model of frictionless matching. Our basic assumptions, including the assumption of fully transferable utility within marriage, are derived from the canonical Becker (1973) model, augmented with matching frictions, in the tradition of Shimer and Smith (2000), but with repeated matching, so that the singles distribution evolves endogenously over time. The problem

[2] We also show in Appendix A that although the war increased the average age of single men, this on its own could not have generated an increase in the female marriage hazard rates, because the number of single women increased even more than the number of older single men.

of finding exogenous variation in sex ratios has been addressed by economists such as Angrist (2002), who used immigration flows, and by Bronson and Mazzocco (2018), who use variation in the size of birth cohorts.

The effect on eventual marital outcomes of war-induced declines in the sex ratio have been studied by Henry (1966), Vincent (1946) and Abramitzky et al. (2011) for the case of World War 1 in France and by Brainerd (2017) for World War 2 in the Soviet Union. Henry (1966) and Abramitzky et al. (2011) both find that lower post-war sex ratios reduce the fraction of women in the affected birth cohorts who ever married, and change the mix of who marries whom. However Brainerd (2017) finds that both men and women are less likely to marry. Bethmann and Kvasnicka (2014) find that low sex ratios increase births to unmarried mothers in Germany after the Second World War[3]. None of these papers are directly concerned with the time it takes to marry, so our analysis of hazard rates and dynamics do not speak to their main results. This distinction is critical in our view, because the fraction of people who eventually marry is a function not only of the hazard rate, but also of the length of time singles spend in matching.

If the heterogeneity in our model were permanent, the environment would be similar to that of Burdett and Coles (1999), who show how composition effects generate multiple steady states in a random-matching model. That sort of multiplicity does not occur in our model, however, due to our reliance on the competitive-search mechanism, similar to the labor-matching model of Shimer (2005), where endogenous surplus allocation ensures uniqueness and Pareto-optimality of the equilibrium. In this sense, our model is closely related to the matching and fertility model of Kennes and Knowles (2015), which builds on elements of Regalia and Ríos-Rull (1999) and Shimer (2005).

The labor-market analog of our argument would be that composition effects can shift the Beveridge curve. Our emphasis on the matching history generating composition effects also

[3]However Henry (1966) and Abramitzky et al. (2011) both find that the lower sex ratio was associated with a smaller post-war age gap between new spouses, a result that our model also generates. In the steady state, our model implies an age difference of 1.79 years, while in the mid 1920s the age difference is on average 1.12 years.

parallels the labor-market competitive-search model of Menzio and Shi (2010); in their analysis, the composition of vacancies shifts over the business-cycle. However, the critical component in their model is heterogeneity that is unobserved at the time of matching, while in our model the heterogeneity is common knowledge during the matching process.

The empirical marriage-matching literature that derives from Choo and Siow (2006) relies on an assumption that is incompatible with our analysis: separability in the marriage-surplus function between observed variables such asage and any other variable excluded from the analysis. This rules out the sort of composition effects we focus on here. The extension of that literature to multi-period matching has also proven very difficult; Choo (2015) appears to be the first paper to achieve this, but is limited to analysis of the stationary state, ruling out the sort of dynamics that we study here.

2 Empirical Analysis

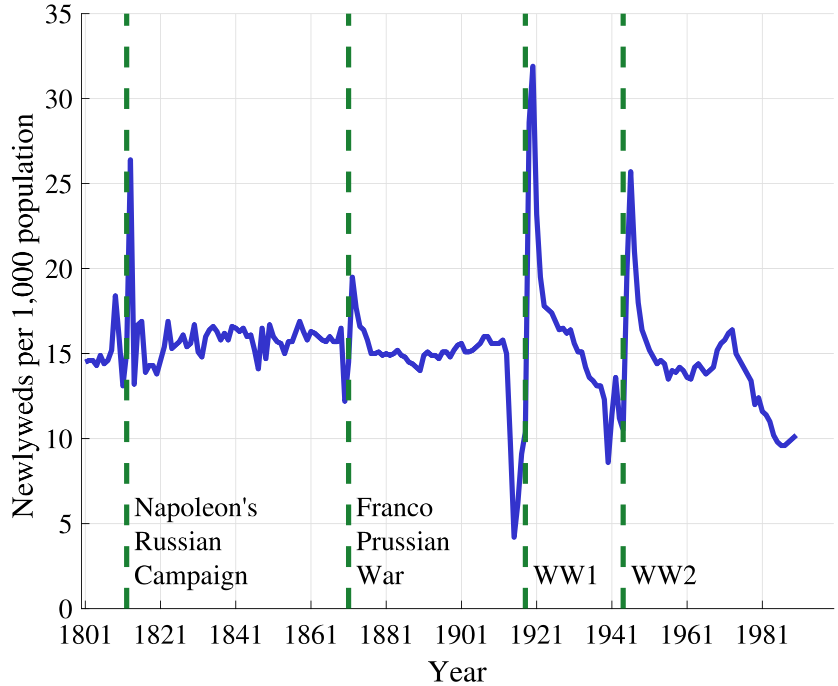
We have three reasons for focusing the analysis below on France in WW1. First, the timing of war with Germany was unexpected, and the impact on civilian life was severe and relatively uniform over the course of the war[4]. Second, the French system of universal conscription dropped all exemptions in 1905, so all men aged 20 and over were subject to military service regardless of marital, parental or economic status. Third, although individual-level records are not available, we have annual Census population statistics for France, by age in years, sex and marital status, starting in 1900; these are not available for the other countries involved in the war.

2.1 Per-capita Marriage Rates

In Figure 1, we see that per-capita marriage rates in France, over the period 1800-1950, followed a pattern of war-time busts followed by post-war booms and then a return to a stationary

[4]World War 2 is much less suited for analyzing the demographic impact of war, because the attack on France was long anticipated, and broke out in the abnormal economic environment caused by the Great Depression, and the abnormal demographic environment caused by World War 1.

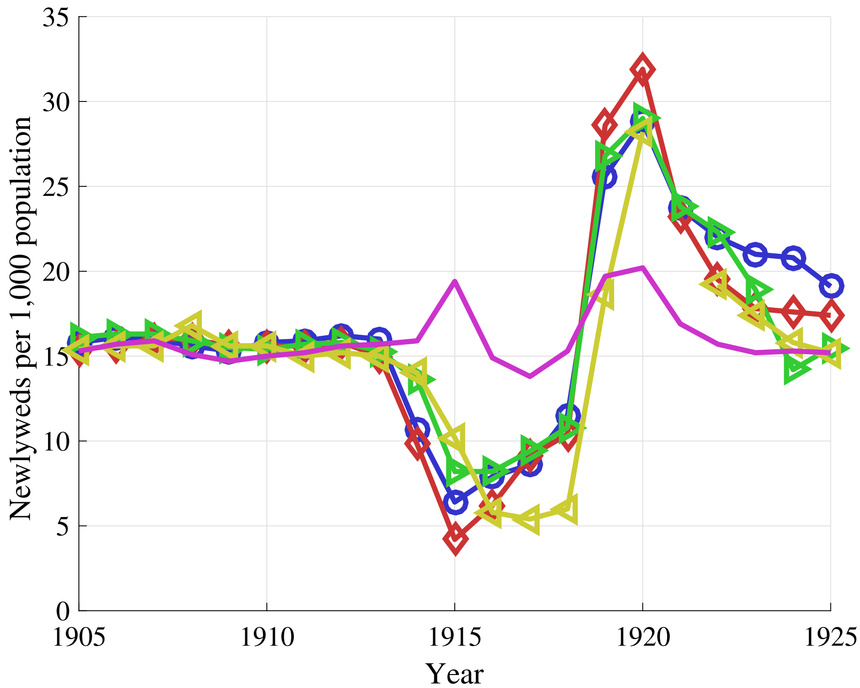
Figure 1: Aggregate Marriage Rate in France



Note: The vertical lines mark the last year of a conflict.
Source: Mitchell (1998).

peace-time baseline. This figure is based on the number of newlyweds per capita, as compiled by Mitchell (1998) from the French Census. We take the per-capita marriage rate to be equal to half the number of newlyweds per capita, as compiled by Mitchell (1998) from the French Census. These cycles are large in amplitude; while the average per-capita marriage rate over time is remarkably stable at 7.5 per thousand, we see it rise by 70% after the Napoleonic wars and after World War 2, by 25% after the much shorter Franco-Prussian war of 1870, and by 100% after WW1. The troughs that correspond with these wars are on the order of 20% below average for the Napoleonic and Franco-Prussian wars, 30% for World War 2, and more than 66% for WW1. The effect of war-time shocks on per-capita marriage rates is also quite persistent. It takes about 5 years after the Franco-Prussian war of 1870 for per-capita marriage rates to return to normal, 10 years after WW1, and 5 years after World War 2. Figure 2 shows that the cycle around WW1 is remarkably similar for other countries: per capita marriage rates in

Figure 2: Aggregate Marriage Rates in Europe



Note: Belgium (○), France (◇), Germany (▷), Italy (◁), England (−).
Source: Mitchell (1998).

Belgium, Germany and Italy all follow the French pattern[5].

The recurrence of this pattern over time and across countries suggests that institutional idiosyncrasies are not the source of the cycle. Furthermore, it would be difficult to reconcile a model of multiple equilibria, given the size of these shocks, with the long-run stability exhibited in Figure 1.

The per-capita marriage rate shown in Figure 1 reflects the sum over all age groups $a \in A$. Let the number of marriages of women of age group a in year t be $M(a,t)$ and the size of the age group be $F(a,t)$. Let the population size be $P(a,t)$. The per-capita marriage rate is given

[5]The pattern for England is quite different; marriage rates increase early in the war, and at no point fall much below the peace-time rate. This may reflect the fact that while Germany, like France, had a pre-existing system of universal conscription, England relied at first on volunteers and when conscription was first introduced, married men were exempted.

by the weighted sum of the "age-group" marriage rates:

$$\frac{\text{Flow of marriages}}{\text{Population stock}} = \frac{\sum_{a\in A} M(a,t)}{P(a,t)} = \frac{1}{P(a,t)} \sum_{a\in A} F(a,t) \times \underbrace{\frac{M(a,t)}{F(a,t)}}_{\text{age-group rate}} .$$

Our analysis will focus on the age-group marriage rate for two age groups: 20-29 years, and 30-39 years; together, these account for more than 80% of female newlyweds and 94% of male newlyweds.

2.2 The Sex Ratio

Huber (1931) reports that military losses (killed and missing in action) amounted to 1.4 million men; this contributed to a severe post-war deficit in the ratio of single men to single women. Using Census data for the peace-time years, we computed the ratio of single men to single women for each age group. In Figure 3, we see that the sex ratio in 1920 is 33% lower for the age 20-29 group than during the pre-war years, while for the age 30-39 group it is 10% lower[6].

2.3 Marriage hazard rates

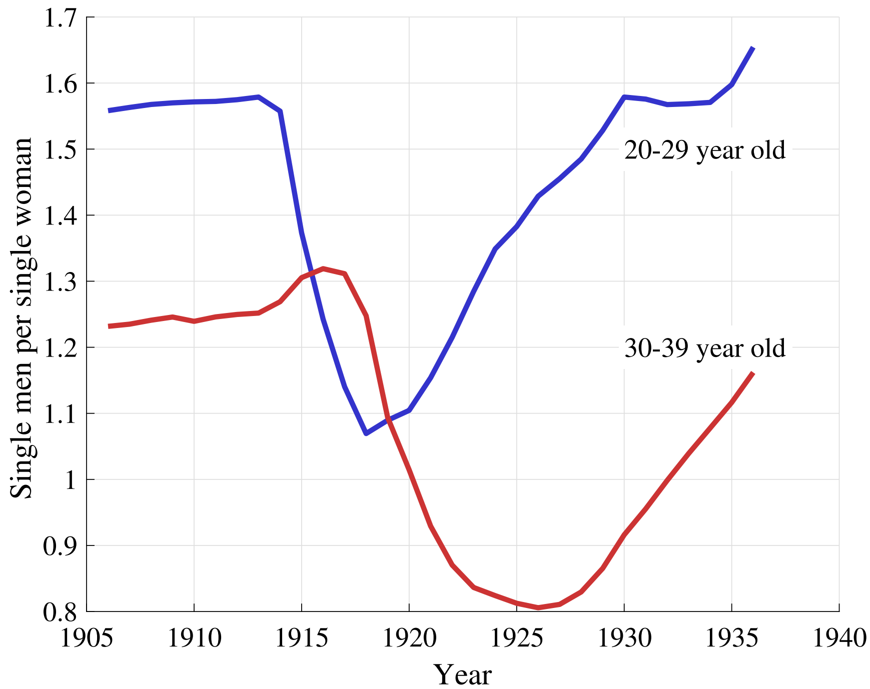
Let the number of single women be $S(a,t)$. The female age-group marriage rate can be decomposed via the following accounting identity:

$$\underbrace{\frac{M(a,t)}{F(a,t)}}_{\text{age-group rate}} = \underbrace{\frac{M(a,t)}{S(a,t)}}_{\text{hazard rate}} \times \underbrace{\frac{S(a,t)}{F(a,t)}}_{\text{single share}},$$

where the hazard rate, the number of matches per single woman, represents the probability that a single woman of age a gets married in a given year. Using this identity, we can compute the hazard rate from observations of the age-group rate and the single share.

[6]The figure also shows the sex ratio for the war years. This was computed by the interpolation method described in Appendix C , exploiting the availability of per-capita marriage rates during the war years.

Figure 3: The Singles Sex Ratio



Source: INSEE and authors' calculations.

The French Census lists population by age, sex and marital status (married, divorced, single, widowed) as of January 1st each year, starting in 1900, with the exception of the war years 1915-1919. In peace time, therefore, we can reconstruct hazard rates from comparing stocks of people by age, sex and marital status in two adjacent years, subject to a few simplifying assumptions. The details of these calculations are in Appendix C.

We apply this method to computing peace-time marriage hazards by sex and age group. Table 1 shows pre- and post war values for the three elements in the accounting equation above. The results reveal three important facts:

1. The post-war time paths of the age-group marriage rates of each age-sex group are similar to those of the per-capita rates in Figure 1. For instance, the age-group rate of men 20-29 almost doubled, from an average of 5.7% before the war to 11.2% in 1919.

2. The hazard rates peak well above their pre-war average in 1919. For men aged 20-29 years,

Table 1: Pre- and Post-War Statistics

	Men		Women	
	20-29	30-39	20-29	30-39
Pre-war average				
Marriage Hazard	0.083	0.094	0.132	0.054
Fraction Single	0.683	0.202	0.427	0.162
Marriage Rate	0.057	0.019	0.056	0.009
Year 1919				
Marriage Hazard	0.157	0.182	0.174	0.085
Fraction Single	0.715	0.226	0.525	0.172
Marriage Rate	0.112	0.041	0.091	0.015
Hazard Share				
	0.932	0.855	0.570	0.876

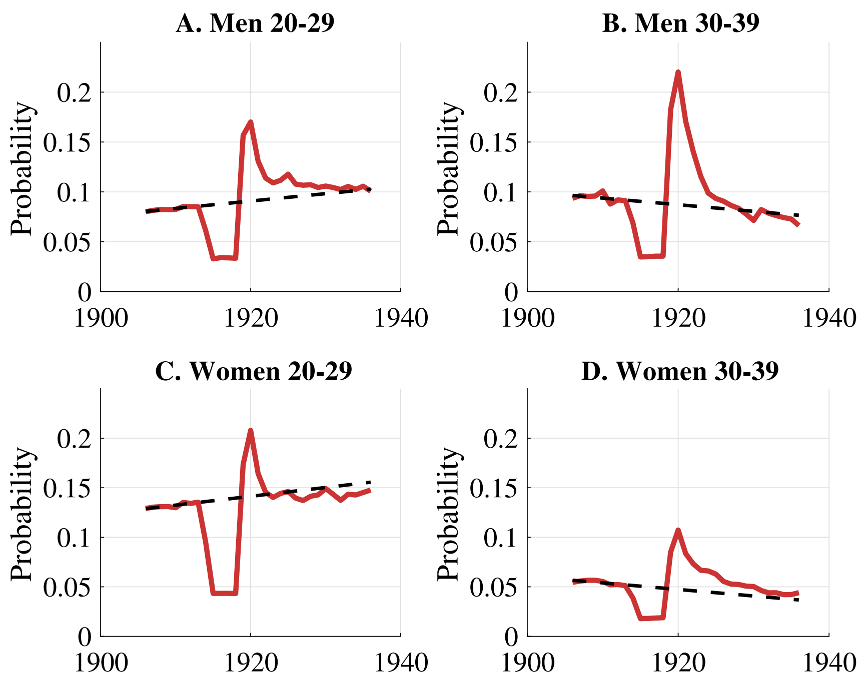
Source: INSEE and authors' calculations.

the increase is 89% above the pre-war average, and for men 30-39 the increase is 94%. For women aged 20-29 and 30-39 years, the peaks are higher by 31% and 57%, respectively.

3. The post-war peaks in the single shares are relatively modest. The hazard rates account for almost all of the variation in the age-group marriage rate of each male age group, 93% and 86% respectively, and for women, 57% and 88%.

We find that the peak in female hazards cannot be explained away by the post-war formalization of war-time matches. In Appendix B we show that the war-time response of the rate of unmarried births is too small to plausibly account for a significant share of the increase in post-war marriages. We also show that the deficit in the number of formal marriages remained large for many years after the war, much longer than would be needed to formalize an existing marriage. In Appendix H we show that a higher post-war fraction of widows cannot account for the rise post-war marriage hazards: previously married women marry at a slightly lower rate than never-married women. In Appendix A we also show that the aging of the male population can not account for higher post-war female marriage hazards.

Figure 4: Marriage Hazard Rates



Note: The solid lines represent the average marriage hazard rate for sex-age group in a given year. The dotted lines represent pre-war trends.
Source: INSEE, Mitchell (1998) and authors' calculations. See Appendix C for details.

It is clear from these results that the hazard rates are crucial for understanding the postwar marriage-rate dynamics; we take this to mean that an individual-level model will be essential for understanding the effect of the war on post-war marriage rates. Furthermore, relative to standard matching models, the correlation of the sex ratio with the female hazards appears to have the wrong sign; we take this to be a compelling reason to study other potential effects of the war.

Figure 4 shows the time series of hazard rates resulting from our calculations; the war-time results are discussed in the next section. Note that it takes until the mid- to late-1920s for female marriage hazard rates to return to their pre-war trends. For men aged 20-29, the marriage hazards remain above trend as late as 1930, 12 years after the war's end. Hence, the disruption of marriage occasioned by WW1 appears to have been very persistent.

2.4 Empirical Impact of the War

We now extend our empirical strategy to war-time marriage and birth hazards, which will be crucial for our model of the post-war transition path: these changes will be used as targets for the calibration of exogenous shocks that represent the effect of the war on the post-war singles pool.

The age-specific data we relied on above is not available for the 1915-1919 interval. Instead, we interpolate using Census data for 1914 and 1920, and per-capita measures of marriages and births for each of the war years. Our method, described in Appendix C, incorporates the peace-time results from section 2, adjusting for the mortality due to the Spanish Influenza in 1918, and for the population increment due to the annexation of Alsace-Lorraine in 1919, as well as the military mortality described below.

Table 2 summarizes the severe war-time declines in marriage and birth hazards by sex and age groups. For both sexes and age groups the war-time marriage hazards fall to about 40% of the pre-war averages: to 47% for men aged 20-29, to 45% for men aged 30-39, and to 45% for women in both age groups. The annual pattern for the war-time marriage hazards was seen above in Figure 4; the key feature is the close resemblance between the war-time patterns of the hazards and the per-capita rates shown in Figure 2, consistent with the previous result that variation in peace-time per-capita rates was mainly due to variation in the hazard rates. Row 2 of the table shows that the birth hazards for married women fell to about 50% of the pre-war average.

We infer from Vallin (1973), who shows the number of dead and disappeared by their year of incorporation into the army, that older soldiers tended to die at much lower rates than young recruits. Figure 5 shows that the men aged 20-25 in 1914 contributed to no less than 5% of the total death toll of the war, while men aged 40-45 contributed to no more than 1% of the total death toll. This is consistent with Figure 3, which showed the decline in the sex ratio (as of 1920) to be largely concentrated in the younger age group.

Table 2: Empirical War Time Changes

	Men		Women	
	20-29	30-39	20-29	30-39
1. Marriage hazard				
War-time avg. / pre-war	0.47	0.45	0.41	0.41
1919 value / pre-war	1.89	1.94	1.31	1.57
2. Birth hazard				
War-time avg. / pre-war	-	-	0.49	0.49

Note: Each statistic represents the ratio of a quantity to its pre-war (1901-1913) average.
Source: INSEE and authors' calculations.

3 The Model

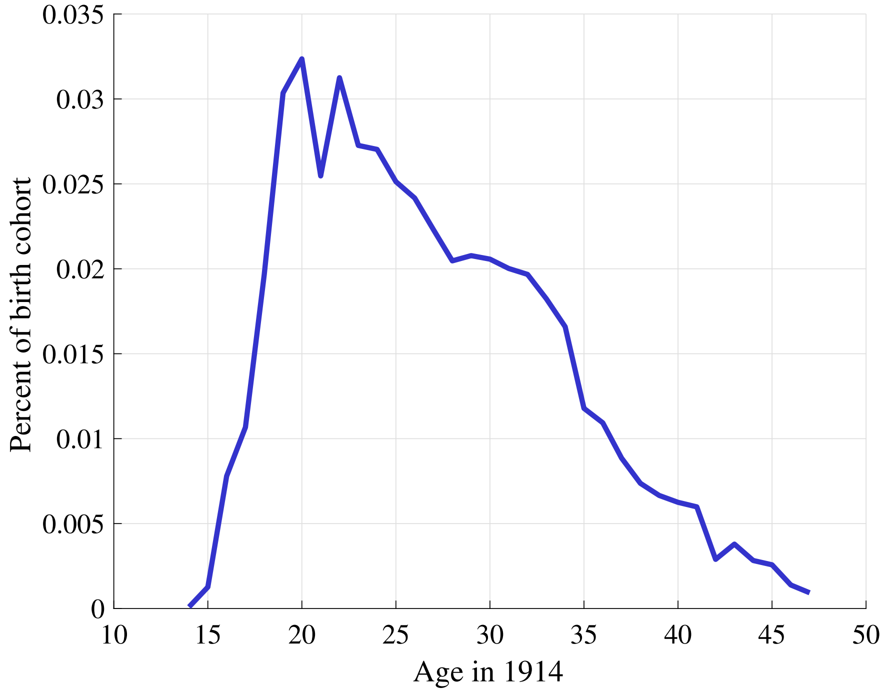
3.1 Demography

Time is represented by an infinite succession of discrete periods. The population is composed of men and women, denoted H and F, respectively. Each period there is an inflow χ_i of new individuals of sex $i \in \{H, F\}$ who begin life as unmatched singles, and age stochastically through 3 stages of life, denoted by $a_i \in \{1, 2, 3\}$; individuals in stage 3 remain there forever. The probability that an individual in stage $a_i < 3$ transitions into stage $a_i + 1$ is denoted by $\delta_i(a_i)$.

Each period, single individuals in stage $a_i < 3$ may be permanently assigned a spouse of the opposite sex, according to a stochastic matching process whose description we defer. Men decide whether to pay a stochastic cost $\xi \in \mathbb{R}_+$ to participate in the matching process[7]. Let $\mu_t(a_H)$ denote the endogenously-determined fraction of stage-a_H men participating in the matching process. We assume that $\xi \in \mathbb{R}_+$ is an *iid* shock realized each period, drawn from the CDF Ξ, before the participation decision.

[7] This feature of the model does not influence the main results. A stochastic entry cost with unbounded support ensures that some males choose not to participate each period, which simplifies the computation of equilibria because men's expected gain from participation is always determined by the indifference of the marginal male. For further discussion of the two types of equilibria that result in the general case, see Wright et al. (2017).

Figure 5: Dead and Disappeared by Birth Cohorts



Note: Military casualties: percent of male birth cohort dead or disappeared, by age in 1914.
Source: Vallin (1973).

The stage of a couple is given by that of the wife, a_F. Married couples are initially childless, but each period, they can choose the probability, π^B, of a birth in the next period, provided that $a_F < 3$ and $k < K$, where K is the maximum number of children. The birth probability generates a disutility $\sigma_F(a_F)\, C\left(\pi^B, a_F, k\right)$, which we interpret as summarizing reproduction-related tradeoffs that we do not model.[8] The probability that a single man in stage a_H dies in a given period is $\sigma_D(a_H)$. We assume, for simplicity, that neither women nor married men face mortality risk.

Single individuals produce a flow of utility equal to $y_i^S(s_i)$ each period, where $i \in \{H, F\}$. Married couples produce utility $y^M(k)$ each period, which is perfectly transferable between

[8]Thus, the fertility-choice aspect of our model should be seen as a reduced form model that cannot distinguish between preference shocks and cost shocks. Shocks to the function C and/or the parameters $\sigma_F(a_F)$ cause variation in the strength of the incentive to have children, which we will refer to for convenience as ”fertility” shocks.

spouses, and is increasing in the number of the couple's children: $y^M(k+1) > y^M(k)$ for $k < K$. For married couples that can have children, the flow utility each period is $y^M(k) - \sigma_F(a_F)\, C\left(\pi^B, a_F, k\right)$. Let β be the discount factor between periods, conditional on survival.

Stochastic aging, an assumption which we borrow from Regalia and Ríos-Rull (1999), is central to our argument; our hypothesis is that the apparent effect of age on marriage probabilities, taken as given in many models, is actually due to heterogeneity in variables that are omitted in the data. In contrast to models that assume directly that gains from marriage are a function of calendar age, in our model the mapping from age to marriage gains will evolve endogenously in response to changes in the composition of the singles pool. Conversely, stochastic aging allows us to track the annual ages of individuals with only a few state variables, making computation and calibration of the model much less cumbersome.

3.2 The value of marriage

Let $Y(a_F, k)$ denote the value of a marriage in state (a_F, k). Since $a_F = 3$ and $k = K$ are absorbing states, the value of a marriage in either of these cases equals the present discounted sum of the utility flow of remaining in the marriage forever: $Y(3,k) = y^M(k) / (1-\beta)$, and $Y(a_F, K) = y^M(K) / (1-\beta)$.

The expected value of the couple with age a_F next period sums over the possible number of kids, weighted by the birth probability π^B:

$$E_{\pi^B}\left[Y(a_F,k)\right] = \pi^B Y(a_F, k+1) + \left(1-\pi^B\right) Y(a_F,k).$$

Thus the expected continuation value sums over the value of each possible age a'_F next period, weighted by the couple's aging probability $\delta_F(a_F)$:

$$E\left[Y(a'_F,k)\,|a_F,k,\pi^B\right] \equiv \delta_F(a_F)\, E_{\pi^B} Y(a_F+1,k) + (1-\delta_F(a_F))\, E_{\pi^B} Y(a_F,k)$$

The utility flow for the couple this period is $y^M(k) - \sigma_F(a_F)\, C(\pi_b, a_F, k)$. The value of a marriage in state (a_F, k) therefore solves the following Bellman equation:

$$Y(a_F, k) = \max_{\pi_B} \left\{ y^M(k) - \sigma_F(a_F)\, C(\pi_b, a_F, k) + \beta E\left[Y(a'_F, k') \,|a_F, k, \pi^B \right] \right\}.$$

The solution defines the optimal decision rule for the birth probability, $\pi_B(a_F, k)$.

3.3 Matching and marriage hazard rates

The key feature of the competitive-search matching process, relative to random search models, is that the surplus allocation in a marriage depends endogenously on the types of the spouses. Thus spouse types are effectively priced by the matching mechanism, which could be described in terms of wage posting in segmented markets, as we do below, following the tradition of Moen (1997) and Shimer (2005), or in terms of second-price auctions, as in Julien et al. (2000), or in terms of contracts, as in Menzio and Shi (2010). Since the Pareto-optimal allocation each period is unique (Shimer, 2005), any Pareto-optimal mechanism will deliver the same equilibrium matching; pecuniary externalities are eliminated, and so multiple equilibria do not arise.

3.3.1 Matching

Matching takes place in separate sub-markets for each type of single women in states $a_F \in \{1, 2\}$. Each period, all single women in states a_F are allocated to their corresponding submarkets, and single men choose which sub-market to search. All of the men in a given sub-market are then assigned randomly to the women. Women who are assigned at least one man choose a husband at random from among their suitors.

Let $P_{i,t}(a_i)$ denote the population of singles of sex $i \in \{H, F\}$ and state a_i at the beginning of period t. Let $N_t(a_F, a_H)$ denote the mass of men in state a_H who participate in sub-market a_F. For each combination of (a_F, a_H), the ratio of men to women in the sub-market, referred to hereafter as the "queue" is given by:

$$\phi_t(a_F, a_H) = \frac{N_t(a_F, a_H)}{P_{F,t}(a_F)}. \tag{1}$$

A standard argument shows that the number of men assigned to each woman follows a Poisson distribution; the probability that there are no suitors in stage a_H for a woman in stage a_F is $e^{-\phi_t(a_F, a_H)}$.

3.3.2 Marriage hazards rates of women

Let $\rho_t(a_F, a_H)$ denote the probability that a woman in stage a_F marries a man in stage a_H. Suppose, for the sake of exposition, that the marriage surplus is higher with stage-2 men than stage-1 men[9]. Optimal matching then implies that women marry a stage-2 man whenever possible. Marriage to a stage-1 man occurs whenever a woman's queue contains at least one stage-1 suitor and no stage-2 suitors. This lexicographic rule (similar to Shi (2002) and Shimer (2005)) implies the following marriage probabilities:

$$\rho_t(a_F, 1) = e^{-\phi_t(a_F, 2)}\left(1 - e^{-\phi_t(a_F, 1)}\right), \tag{2}$$

$$\rho_t(a_F, 2) = 1 - e^{-\phi_t(a_F, 2)}. \tag{3}$$

The total number of marriages by a_F-women is $P_t(a_F)\sum_{a_H} \rho_t(a_F, a_H)$. Thus, the marriage hazard rate of these women is equal to

$$\pi_{F,t}(a_F) = \sum_{a_H} \rho_t(a_F, a_H). \tag{4}$$

[9]In fact, this will turn out to be the case in our calibration results. However our model accommodates both cases.

3.3.3 Marriage hazards rates of men

The number of marriages between men in stage a_H and women in stage a_F equals $\rho_t\left(a_F, a_H\right) P_{F,t}\left(a_F\right)$. This implies that the marriage probability of stage-a_H men in sub-market a_F is:

$$\frac{\rho_t\left(a_F, a_H\right) P_{F,t}\left(a_F\right)}{N_t\left(a_F, a_H\right)} = \frac{\rho_t(a_F, a_H)}{\phi_t(a_F, a_H)}.$$

If a woman has more than one suitor of the type she is to marry, each faces equal probability of being selected. The probability that a man in stage a_H marries, conditional on participating in the matching process, is therefore given by:

$$\pi_{H,t}\left(a_H\right) = \frac{1}{\sum_{a_F} N_t\left(a_F, a_H\right)} \sum_{a_F} N_t\left(a_F, a_H\right) \frac{\rho_t(a_F, a_H)}{\phi_t(a_F, a_H)}. \tag{5}$$

3.4 Value functions of single agents

Let $V_{F,t}\left(a_F\right)$ denote the value of a woman in stage a_F at the beginning of period t, and $R_{F,t}\left(a_F\right)$ her value of remaining single at the end of the period; the difference defines $v_{F,t}\left(a_F\right)$, the expected gain from marrying during the period:

$$V_{F,t}\left(a_F\right) = R_{F,t}\left(a_F\right) + v_{F,t}\left(a_F\right). \tag{6}$$

If $a_F < 3$, the value of remaining single at the end of the current period comprises the utility flow from being a single, and a discounted, expected continuation value that depends upon the probability of transitioning into the next stage:

$$R_{F,t}\left(a_F\right) = y_F^S\left(a_F\right) + \beta\left[\left(1 - \delta_F\left(a_F\right)\right) V_{F,t+1}\left(a_F\right) + \delta_F\left(a_F\right) V_{F,t+1}\left(a_F + 1\right)\right]. \tag{7}$$

Note that $R_{F,t}\left(3\right) = V_{F,t}\left(3\right) = y_F^S\left(3\right) / \left(1 - \beta\right)$, since we assume that woman in stage 3 do not participate in the marriage market, We define similar value functions for men in stage a_H, conditional on participation: $V_{H,t}(a_H)$ denotes the pre-matching value, $R_{H,t}(a_H)$ the value at the end of the period, and $v_{H,t}(a_H)$ the expected gain from marrying, so that:

$$V_{H,t}(a_H) = R_{H,t}(a_H) + v_{H,t}(a_H). \tag{8}$$

Men in stage a_H participate if and only if their expected gain is at least as large as the cost of participating: $\xi \leq v_{H,t}(a_H)$. Thus, the probability that a man in stage a_H participates is $\mu_t(a_H) = \Xi(\xi_t^*(a_H))$, where $\xi_t^*(a_H) = v_{H,t}(a_H)$ denotes the threshold value. Since ξ is *iid*, $\mu_t(a_H)$ is also the proportion of these men participating in the marriage market at date t.

Before the realization of his participation cost ξ_{it}, the *ex-ante* value for each man i in period t is:

$$W_{H,t}(a_H) = (1-\mu_t(a_H))\, R_{H,t}(a_H) + \mu_t(a_H)\, E\left[V_{H,t}(a_H) - \xi_{it} \,|\xi_{it} \leq \xi_t^*(a_H)\right]. \tag{9}$$

The value of remaining single at the end of the period is then:

$$R_{H,t} = y_H^S(a_H) + \beta\left[(1-\delta_H(a_H))\, W_{H,t+1}(a_H) + \delta_H(a_H)\, W_{H,t+1}(a_H+1)\right], \tag{10}$$

where $W_{H,t}(3) = y_H^S(3)\,/\,(1-\beta)$. That is, the *ex-ante* value of being single in stage 3 is the value of remaining a single man forever.

3.5 Laws of motion

The number of single men in stage 1 at date $t+1$ equals the number of date-t single men in stage 1 who did not transition into stage 2, did not marry, and did not die, plus the flow of new stage 1 men who did not die:

$$(11) \quad P_{H,t+1}(1) = (1 - \delta_H(1))(1 - \sigma_D(1))(1 - \mu_t(1)\,\pi_{H,t}(1))\,P_{H,t}(1) + \chi_H(1 - \sigma_D(1)),$$

The number of single men in stage 2 at date $t+1$ equals the number of single men in stage 2 who did not transition into stage 3, did marry and did not die; plus single men in stage 1 at date t who transitioned into stage 2 but did not marry and did not die:

$$(12) \quad P_{H,t+1}(2) = (1 - \delta_H(2))(1 - \sigma_D(2))(1 - \pi_{H,t}\mu_t)\,P_{H,t}(2) + \delta_H(1)(1 - \sigma_D(1))(1 - \mu_t(1)\,\pi_{H,t}(1))\,P_{H,t}(1).$$

For single women in stage 1, the corresponding equations are:

$$(13) \quad P_{F,t+1}(1) = (1 - \delta_F(1))(1 - \pi_{F,t}(1))\,P_{F,t}(1) + \chi_F,$$

and for single women in stage 2

$$(14) \quad P_{F,t+1}(2) = (1 - \delta_F(2))(1 - \pi_{F,t}(2))\,P_{F,t}(2) + \delta_F(1)(1 - \pi_{F,t}(1))\,P_{F,t}(1).$$

3.6 Utility Posting

In each sub-market a_F the women post a utility offer $w_t(a_F, a_H)$ for men in stage a_H. The utility offers are common knowledge and women commit to their offers. The optimal offer $w_t(a_F, a_H)$ by women in stage a_F maximizes the woman's expected gain from marrying, subject to the constraint that participating men must receive their expected values.

The expected gain from marriage for a man in this sub-market is

$$\tilde{v}_{H,t}(a_F, a_H) = \frac{\rho_t(a_F, a_H)}{\phi_t(a_F, a_H)} w_t(a_F, a_H).$$

If participating men in a given state a_H enter both markets, they must be indifferent between the two sub-markets:

$$v_{H,t}(a_H) = \tilde{v}_{H,t}(1, a_H) = \tilde{v}_{H,t}(2, a_H). \tag{15}$$

For women, the expected gain from marriage $v_{F,t}(a_F)$ is defined by the following optimization problem

$$v_{F,t}(a_F) = \max_{w_t(a_F,a_H)} \sum_{a_H} \rho_t(a_F, a_H)\left[x_t(a_F, a_H) - w_t(a_F, a_H)\right] \tag{16}$$

$$s.t. \quad v_{H,t}(a_H) = \frac{\rho_t(a_F, a_H)}{\phi_t(a_F, a_H)} w_t(a_F, a_H), \tag{17}$$

where the surplus $x_t(a_F, a_H)$ is defined by the output of a marriage, net of the reservation values of the husband and the wife:

$$x_t(a_F, a_H) = Y(a_F, 0) - R_{H,t}(a_H) - R_{F,t}(a_F). \tag{18}$$

3.7 Equilibrium

A stationary competitive-search equilibrium is a list of equilibrium objects, constant over time:

$$\{\phi(a_F, a_H), \xi^*(a_H), v_H(a_H), v_F(a_F), R_H(a_H), R_F(a_F), P_H(a_H), P_F(a_F)\}. \tag{19}$$

The list comprises queue lengths, cost thresholds, expected values of marriage, reservation values, and single populations sizes. The following conditions must be satisfied. First, given the reservation values, the surplus is defined by Equation (18). Second, women optimize taking men's value of marriage, $v_H(a_H)$, as given. Thus, Equations (16)-(17) are satisfied. Men optimize as well, hence the marginal man is indifferent between participating or not, i.e., $\xi^*(a_H) = v_H(a_H)$, and Equation (15) is also satisfied where applicable. Third, the allocation of participating men to sub-markets is feasible, i.e.,

$$\sum_{a_F} \phi(a_F, a_H) P_F(a_F) \leq \Xi(\xi^*(a_H)) P_H(a_H). \tag{20}$$

Fourth, the reservation values of men and women satisfy Equations (8)-(10) and Equations (6)-(7). Finally, the population of single satisfy Equations (11)-(14).

For the analysis of transition paths in Section 5, a more general definition of the dynamic equilibrium is required to allow for non-stationarity; this is described in Appendix D.

4 Steady-state analysis

In this section, we calibrate the model so that the stationary equilibrium matches empirical targets drawn from French pre-war statistics. Since the unobserved heterogeneity in the model is age-related, we focus on age profiles of marriage and birth hazard rates as our empirical targets, as well as the sex ratio of singles.

To interpret the model's results in terms of age profiles, we impose that new agents in our model are 18 years old when they first enter the singles pool, and that a model period lasts a year. The "calendar age" of an agent is thus given by 18 plus the number of years since s/he was a new agent, and is denoted by τ.

In the stationary equilibrium, there are four possible states a man can be in: single in stage 1, 2 or 3 and married. Let

$$\lambda_H^\tau = \left[\lambda_{H,1}^\tau, \lambda_{H,2}^\tau, \lambda_{H,3}^\tau, \lambda_{H,4}^\tau\right]$$

represent the mass of men in each one of these states, at calendar age τ, in the steady state. By construction, the initial condition is $\lambda_H^{18} = [\chi_H, 0, 0, 0]$. Then evolution of λ_H^τ then follows a linear process given by the equilibrium transition matrix T_H, defined in Appendix E:

$$\lambda_H^{\tau+1} = \lambda_H^\tau T_H.$$

Once we have the values for $\{\lambda_H^\tau\}_{\tau=18}^{\tau=39}$ we can easily compute statistics for men for each calendar-age $\tau \geq 18$, including the distribution over marital state and the marriage hazard. The logic is also the same for women: we derive statistics for each age given a female transition matrix T_F and distribution vector λ_F^τ:

$$\lambda_F^{\tau+1} = \lambda_F^\tau T_F.$$

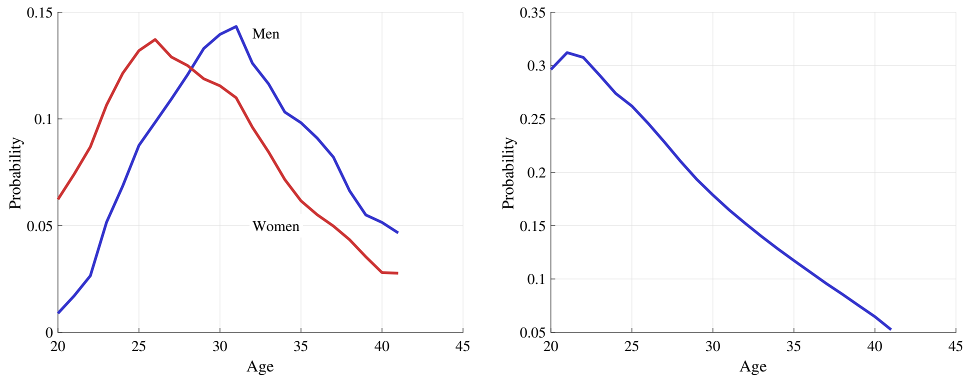
Note that λ_F^τ and T_F are considerably larger than λ_H^τ and T_H, as there are more female states the we must track: women may be single or married in stage 1, 2 or 3, and, when married, they may have 0, 1, 2 or 3 children.

At each calendar age τ, the age-specific distributions for sex i are dictated by the rates $\delta_i\left(a\right)$ at which agents transition from one stage to the next. Matching the height of the age profiles of marriage hazard restricts the parameters governing the value of single life, $y_i^S\left(a_i\right)$. Matching the slope restricts, in turn, the transition rates between stages, $\delta_i\left(a_i\right)$. Since, in addition, the female stage accounts for all age-related birth-hazard variation, the birth-hazard profiles pin down the fertility differences by stage, $\gamma_k\left(k\right)$ and $\gamma_a\left(a_F\right)$. In this way the heterogeneity in the model is pinned down by the pre-war empirical targets.

The computation of birth hazards is analogous to that of marriage hazards, as described in section 2. The number of live births, by age and year and mother's marital status, is listed in France's Vital Statistics starting in 1901[10]. For each age-year group, we set the married-birth

[10]Source: INSEE, Etat civil.

Figure 6: Pre-War Age Profiles



A. Marriage Hazards

B. Birth Hazards of Married Women

Note: Marriage hazard rates (panel A) and birth hazard rates of married women (panel B) by age, averaged over 1901-1913.
Source: INSEE and author's calculations.

hazard to the number of married births divided by the number of married women as of the start of each year, obtained from the Census, as described in section 2.

Panel A of Figure 6 plots the marriage-hazard rates by sex and age, and Panel B plots the birth hazard for married women, both averaged over the pre-war years. These profiles, along with that of the sex ratio, constitute the principal calibration targets for the model in the quantitative analysis below. The critical features are that:

1. Men tend to marry later than women: the peaks in marriage-hazard rates, at about 14% for each sex, occur around 24 years old for women, and around 28 years for men.

2. The married-births-hazards grow with age at first, peaking at 30% when women are relatively young, about 22 years old, and decline uniformly thereafter, to about 5% for women in their late 30s.

4.1 Functional forms

The output of marriage in our calibration is characterized by diminishing marginal returns to children, with a parameter ω that can be adjusted to vary the intensity of preferences over children:

$$y^M(k) = \omega \ln(1+k)\,. \tag{21}$$

The birth-probability cost function has two free parameters for each state (a_F, k) in which women can give birth, $\gamma_a(a_F)$ and $\gamma_k(k)$:

$$\sigma_F(a_F)\,C(\pi, a_F, k) = \sigma_F(a_F)\,\gamma_k(k)\,\pi^{\gamma_a(a_F)}. \tag{22}$$

The utility of single life is set to a separate value for each single state (i, a_{it}). The distribution of the male participation cost is assumed to be log-normal, so that $\ln(\xi) \sim N(\mu_\xi, \sigma_\xi)$. More general functional forms, such as a beta distribution, could be used without affecting the main results, provided that the support is unbounded.

4.2 Parameter Selection

Some of the parameters are set to values outside the calibration loop: the annual discount factor is set to $\beta = 0.96$, a standard value in the macro literature; the maximum number of children per family to $K = 3$; and the inflow rate of men to $\chi_H = 1$, a normalization. The set of remaining parameters is denoted by Θ:

$$\Theta = \left(\omega, \gamma_k(k), \gamma_a(a_F), \mu_\xi, \sigma_\xi, \delta_F(a_F), \delta_H(a_H), y_F^S(a_F), y_H^S(a_H), \chi_F\right).$$

Let $m_i^\tau(\Theta)$ and $b^\tau(\Theta)$ represent the model's steady-state marriage and birth hazards (respectively) for people of calendar age τ and sex i, and let $SR^\tau(\Theta)$ be the corresponding sex ratio.

The corresponding empirical target values are denoted by bold face. The values of the free parameters are set to minimize the Euclidean score function:

$$\min_{\Theta}\left\{\sum_{\tau=20}^{45}\sum_{i=H,F}\left(m_i^{\tau}\left(\Theta\right)-\mathbf{m}_i^{\tau}\right)^2+\sum_{\tau=20}^{45}\left(b^{\tau}\left(\Theta\right)-\mathbf{b}^{\tau}\right)^2+\sum_{\tau=20}^{45}\left(SR^{\tau}\left(\Theta\right)-\mathbf{SR}^{\tau}\right)^2\right\},$$

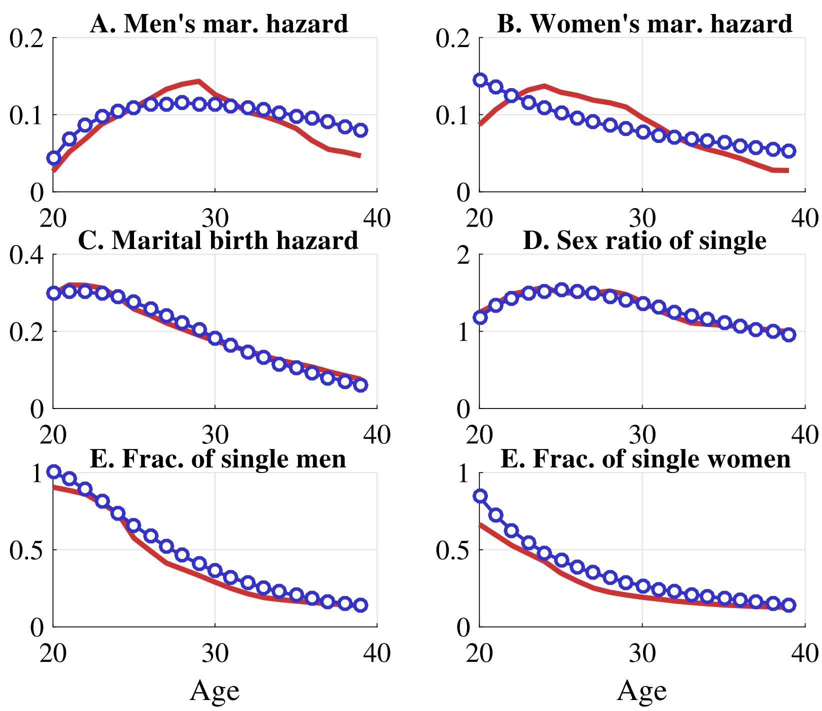
Figure 7 shows that the age profiles implied by the model match the two critical features of the empirical marriage hazard profile : it is lower for men in their early 20s than for men in their late 20s, and it is higher for women in their early 20s than for women in their late 20s[11]. The model achieves this by imposing a relatively high utility of single life for women with $a_F = 2$, and a relatively low utility of single life for men with $a_H = 2$. For men, $y_H^S\left(1\right) = 16.52$, while $y_H^S\left(2\right) = 0.79$, but for women, $y_F^S\left(1\right) = -.02$, while $y_F^S\left(2\right) = 1.45$. The transition rates between life stages are similar for both sexes: roughly $\delta\left(1\right)$ =0.14 and $\delta\left(2\right)$ =0.02, so that young people age quickly into the middle stage, and remain there for a relatively long time. The complete list of parameter values is shown in Table 6 in Appendix F.

5 The Impact of the War

This section describes the baseline experiment, which incorporates the war into our equilibrium analysis by subjecting the calibrated model to exogenous military-mortality and fertility shocks. We start from the steady-state equilibrium and assume the economy is hit by two unanticipated shocks, and compute the perfect-foresight transition path back to the steady state. We also assume, throughout the experiments, that the sex ratio of entrants into the single pool follows the empirical age-18 sex ratio from the Census, shifting exogenously until 1940. The perfect foresight assumption implies that agents are aware of all current and future equilibrium variables throughout the transition, from the arrival of the initial (unanticipated) shocks until the economy

[11] An improved fit for marriage hazards of the young agents could be obtained by increasing the number of life stages, but as the age profile for the sex ratio of singles is almost exactly matched, the additional complication appears unjustified.

Figure 7: Baseline Calibration: Age Profiles



Note: Marriage hazard rates equal the number of marriages per year per single person; sex ratio equal to number of single men per single woman. Frac. of Singles equals the ratio of singles in the age-sex group to the total population of that group; data (−) and model (∘).

Source: INSEE and authors' calculations.

Table 3: The Baseline Experiment

	Marriage hazard					
	Men		Women		Birth hazard	
	20-29	30-39	20-29	30-39	20-29	30-39
War-time decline (model/data)	1.09	1.01	0.99	1.04	1.09	1.01
1919 peak (model/data)	0.42	0.61	0.66	0.74	0.12	0.29
1920 peak (model/data)	0.21	0.36	0.18	0.26		
Months to 50% of peak (model/data)	0.52	1.01	0.38	0.38		

Note: Statistics computed from calibrated model, relative to empirical analogs from Table 1.

arrives back at the steady state[12].

5.1 The Shocks

The male-specific mortality parameter, $\sigma_D(a_H)$, and the fertility cost parameter $\sigma_F(a_F)$ jump to their war-time values in the first period of the transition and remain at these values for 4 more periods; they and return to their pre-war values in period 6.

We choose the war-time male-specific mortality to match the data on military casualties discussed in Section 2 (Figure 5). This turns out to imply that the death toll in the model is concentrated on men in stage $a_H = 1$, so we set $\sigma_D(1) = 0.075$, and $\sigma_D(2) = 0$. We choose the two fertility shocks to minimize the Euclidean distance between the model-generated average war-time decline in marriage hazard rates for the four age-sex groups, and their empirical counterparts. This requires $\sigma_F(1) = 8.25$, and $\sigma_F(2) = 5.25$.

5.2 War-time Results

The first row of Table 3 reports the decline in marriage and birth hazards in the model relative to the decline measured in the data (reported in the first part of Table 2). Take the case of 20-29 year-old men for instance. In the model, their marriage hazard declines, on average, to 42% of its steady state value during the war. In the data the decline is slightly smaller, as the first part of Table 2 indicates that the marriage hazard declines to 47% of its pre-war average. The model result therefore exceeds the actual decline, but by less than 10% of the empirical change. The remainder of the first row of Table 3 shows that the model-generated declines in average marriage and birth hazards, across sex and age groups, are much closer to the actual declines. This should not be taken however as a metric of the model's performance; the key result of this exercise is that we have disciplined the magnitude of the shocks that will drive the post-war dynamics.

5.3 The Post-war Transition Path

Figure 8 compares the predicted and empirical transition paths of average marriage hazards by sex and age groups. As in the data, the model generates large and persistent peaks in these hazard rates. The female marriage hazards in the model rise by 20% and 42% for the younger and older age groups respectively, despite the severe decline in the sex ratio in each of these age groups[13].

To understand the post-war response of marriage hazards, consider the how the post-war distribution of singles deviates from the steady state. Figure 9 shows that the war-time marriage bust results in a relative abundance of stage-2 singles of both sexes. Because the calibration implies that aging reduces the marriage propensity for women but increases it for men, this has conflicting implications for the post-war marriage hazards of single men and women. We

[12] The solution method used to compute the transition path is described in Appendix D.

[13] The timing of the peak is a year earlier in the model than in the data; this year-long delay could be driven by higher matching frictions due to the chaos of demobilization that is not modeled here.

can distinguish three separate composition effects in the diagram: the post-war abundance of stage-2 men as a share of single men (panel A), the post-war abundance of stage-2 women as a share of single women (panel B), and the post-war abundance of stage-2 men per single woman (panel C).

It is the third effect that turns out to be critical; this makes it easier for women to find a match and so raises the marriage hazards of both stage-1 and stage-2 women, generating the post-war peak in female marriage hazards. On its own, the second effect would cause post-war marriage rates, both male and female, to fall below the steady state[14]. Our calibration implies that the third effect is larger than this second effect, and so women's marriage hazards rise sharply when the war ends.

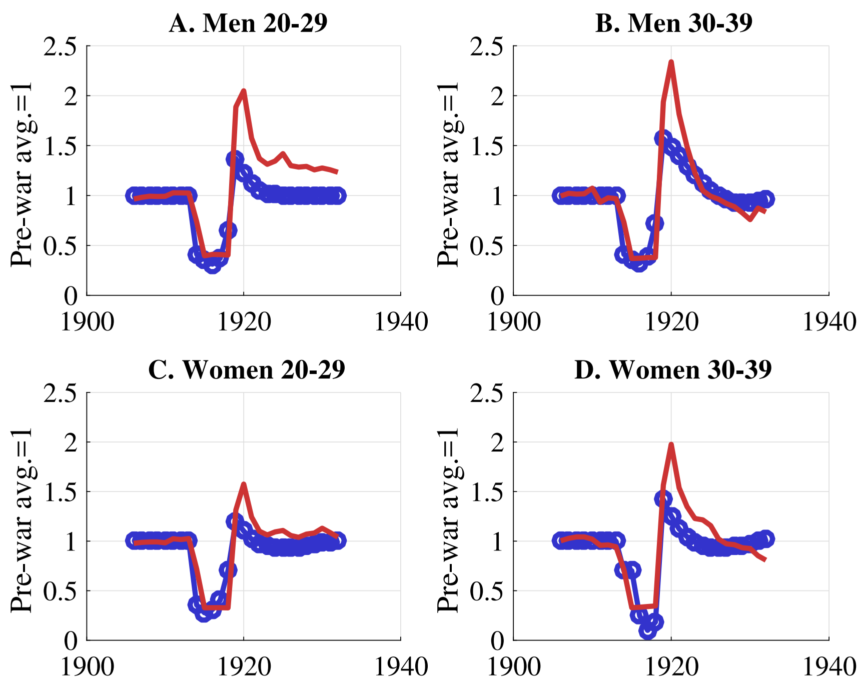
The first effect causes post-war male marriage hazards to rise above the steady-state rate. This is reinforced by two empirical features of the male age profiles: the military-death profile forces all of the model mortality onto the stage-1 men, and the marriage-hazard profile forces the marriage rate of the stage-1 men to zero. The first effect is also stronger than the second effect, accounting for the peak in men's marriage hazards after the war.

The model does more than explain the qualitative responses. Table 3 shows that the female marriage-hazards peaks generated by the model account for most of the peaks in the empirical hazards: 66% for the 20-29 year-old age group and 74% in the age 30-39 group. For men, the model also accounts for a significant, but more modest share. In the case of 20-29 year-old men for instance, the empirical marriage hazard peaks at 89% above its pre-war value in 1919—see Table 2. The model implies a peak at 37% above the steady state. Hence, the baseline model accounts for 37/89 = 42% of the post-war peak for these men. Similarly, the model accounts for 61% of the peak for men in the older group[15].

[14]This is due to the fact that the war-time marriage rate drops much further (to 17% of its steady-state value) for stage-2 women than for stage-1 women (to 42%). This asymmetry arises because because stage-2 women, having a higher autarky value than stage-1 women, marry mainly for the chance to raise children, so the value of marriage for these women is much more reduced by the war-time fertility bust.

[15]In Appendix G we show that, holding the calibration fixed, the empirical variation in the initial (Age-18) sex ratio accounts for a small but significant share (about 18%) of the post-war increase in the female marriage

Figure 8: Marriage Hazards During the Transition

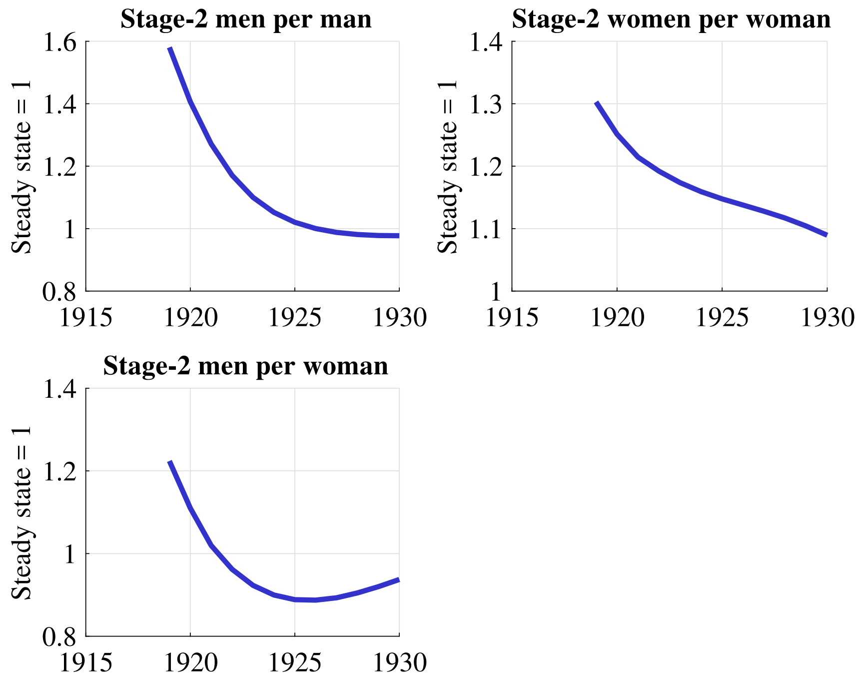


Note: Marriage hazards on the transition path, relative to their pre-war average; data (−) and model (∘). Source: INSEE, Mitchell (1998) and authors' calculations.

The last row of Table 3 indicates the model's ability to reproduce the persistence of the war-time shock. At first, a portion of the post-war peaks dissipates fairly quickly. Consider the half-life of the marriage-rate peaks, defined as the time it takes for the gap between the post-war peak and the pre-war averages to fall to 1/2 of its peak in 1919-20; by analogy, we define the "quarter-life" as the time it takes for the gap to fall to 1/4 of its peak. For the marriage hazard of 20-29 year-old men, the observed half-life is about 32 months. The remainder persists much longer; as late as 1930, more than 25% of the hazard deviation remains. In the model, the half-life is about 17 months, about 52% of that in the data, and the quarter-life is 30 months. The reason for this persistence is that the $a_H = 2$ share of single males for 1930, as shown in Figure 9.A, remains well above the steady-state level. For 30-39 year-old men, the model generates a half-life almost identical (1% higher) to what is measured in the data. For women, the model

hazard. While we take this as exogenous variation in the current analysis, we found that recalibration of the model with this variation excluded yielded similar results to our baseline calibration.

Figure 9: Post-war Single Population



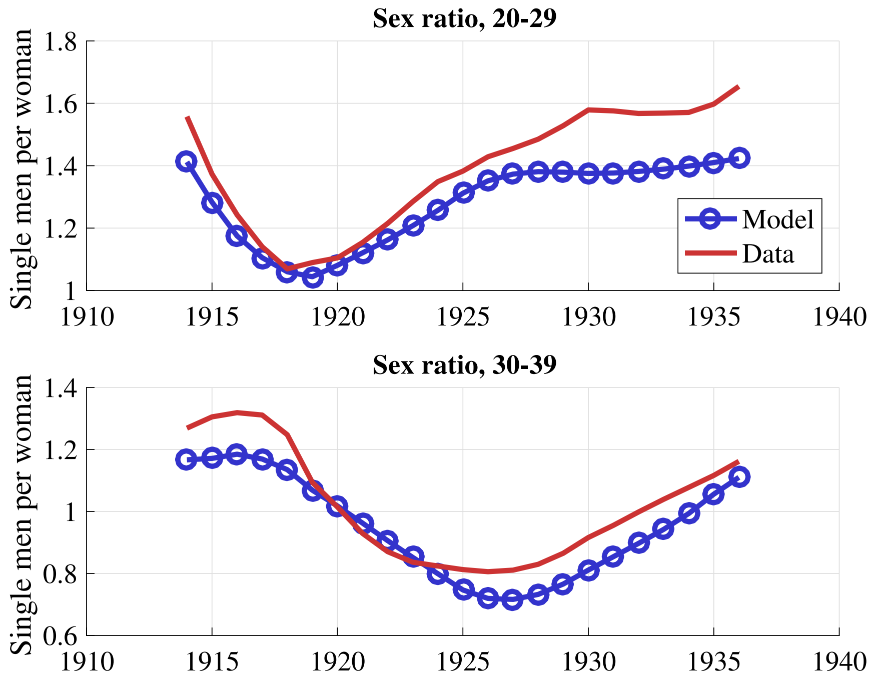
Note: Post-war singles population in the baseline model, relative to the stationary equilibrium.

generates 38% of the observed half-life for both age groups, although the persistence in the data is more modest. The baseline model therefore succeeds in generating substantial persistence.

The average age of women at marriage increased in France in the aftermath of the war, as documented by previous empirical work, such as Henry (1966) and Abramitzky et al. (2011). In the model, the average age also increases: the pre-war average is 25 years old, the post war average about 25.6 years; the age gap for new marriages in the year 1924 in the model is lower by 6 months, a decline of about 25% relative to the steady-state.

The most significant gap between model and data is in the post-war response of the marital birth rate. Table 3 shows that the 1919 peak generated by the model accounts for only 12% of the observed peak for the 20-29 year-old married women, and 29% for the 30-39 year-old married women. Stage-1 women, who, as noted earlier, have a lower birth hazard than stage-2 women, account for a higher share of marriages after the war, further reducing the post-war birth-rate

Figure 10: Sex-Ratio Dynamics



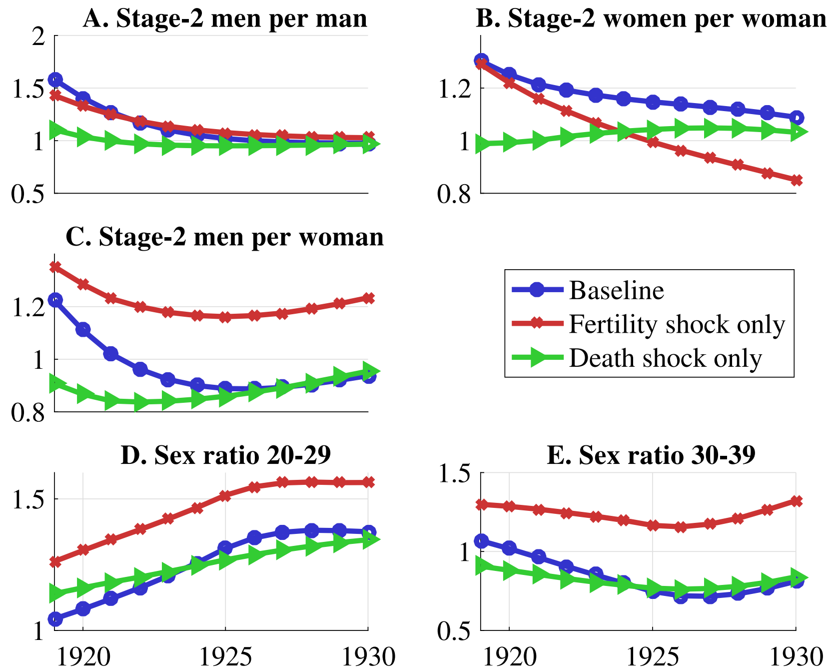
Note: Sex ratios (single men per single women) on the transition path: baseline model v. Census statistics. Source: INSEE and authors' calculations.

response.

Figure 10 shows the sex ratio of singles, by age group. We note that the model reproduces well the decline in the sex ratio of single as well as its recovery until at least the middle of the 1920s. This implies that the model results are not driven by unrealistic movements of the post-war sex ratio.

The cause of the war-time marriage bust is not the subject of our paper. Yet, it is interesting to see that the two shocks described earlier, i.e. the male-mortality shock and the fertility shock, are sufficient to explain all the war-time decline in marriage and married births, with two simple shocks whose magnitudes can be set directly from empirical observations. Of course this baseline exercise does not explain all of the post-war female marriage-hazard peaks, but we will return to this in a later section where we consider another demographic shock, to post-war birth incentives.

Figure 11: Transition-Path Experiments



Note: Panels A, B and C: ratio of stage-2 singles to singles population, normalized to steady state value; Panels D and E: single men per single woman.

6 Mortality vs. Fertility Shocks

To assess the contribution of each shock individually, we present the results of two variations on the baseline experiment with the calibrated model: the fertility-shock experiment and the mortality-shock experiment. Each consists of recomputing the transition path with only one of the two calibrated shocks from the baseline experiment. Figure 11 compares the behavior of the composition of the singles population in the baseline and in each experiment.

Consider first the size of the marriage-hazards peaks in 1919. The first three panels of Figure 11 show that the fertility-shock results are very close to the baseline results: all of the ratios of stage-2 singles to singles population are much further from baseline for the case of the mortality-shock series. Despite the post-war increase in the calendar age of single males, the mortality shock fails to generate *any* increase in the proportion of stage-2 singles relative to same sex; in particular, the number of stage-2 men per single woman remains *below* its steady state value in 1919. This implies in turn a lower matching rate for women after the war, the opposite of the empirical pattern described in Section 5.3.

Table 4 summarizes the quantitative results of the two experiments. The male-mortality experiment implies significant declines of the marriage hazards of women in 1919; their marriage hazards now *drop* by a magnitude equivalent to a fifth of the actual peak. This confirms that the mortality shock in our model is very strong, as suggested by previous work on the sex ratio, such as Abramitzky et al. (2011). Indeed the last two panels of Figure 11 show that the mortality shocks generate virtually all of post-war variation in the sex ratio. Thus we have not assumed away the impact of the sex ratio; the positive composition effects that generate the post-war peaks in the female marriage rates are simply much stronger than the negative effect of the mortality shocks. From this, we conclude that, even though male mortality is the key to the changing sex ratio of singles, the role of the sex ratio is relatively modest in the determination of the post-war marriage hazards.

The last two columns of Table 4 reveal that, as with marriage rates, post-war birth hazards are reacting to the war-time fertility shock much more than to the male-mortality shock. As in the data, the overall response is positive, but the post-war fertility peaks remain relatively small: 13% of the actual percent change for 20-29 year-old married women, and 37% for 30-39 year-old married women. This suggests the possibility that a shock to post war birth incentives may have caused part of the increase in marriage hazards.

7 Post-War Birth Rates

The politics of post-war France was transformed by a strong pro-natalist movement arising from the fear that low fertility was jeopardizing France's military strength (see Roberts, 1994 and Robert, 2005). By 1920, the pro-natalist movement was sufficiently powerful that the Conservative government enacted significant financial incentives to increase birth rates, as well

Table 4: Experiments

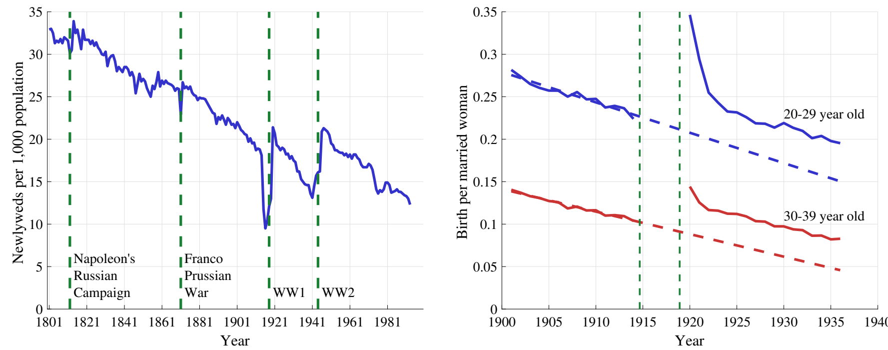
	Marriage hazard					
	Men		Women		Birth hazard	
	20-29	30-39	20-29	30-39	20-29	30-39
War-time decline (model/data)						
Baseline	1.09	1.01	0.99	1.04	1.09	1.01
Fertility shock	1.14	1.05	0.98	0.97	1.09	1.00
Mortality shock	-0.11	-0.11	0.00	0.02	-0.00	-0.00
1919 peak (model/data)						
Baseline	0.42	0.61	0.66	0.74	0.12	0.29
Fertility shock	0.26	0.41	0.98	1.10	0.13	0.37
Mortality shock	0.11	0.15	-0.22	-0.22	0.01	0.10
Months to 50% of peak (model/data)						
Baseline	0.52	1.01	0.38	0.38	-	-
Fertility shock	0.55	0.95	1.63	1.53	-	-
Mortality shock	0.37	0.79	-	-	-	-

as legislation against abortion and contraception, and employers were pressured to offer family allowances to workers with more than two children[16].

During this period, the birth rate to married mothers rose to well above trend and remained high throughout the 1920s. The per-capita birth rate in panel A of Figure 12, as computed from Mitchell (1998), reveals a war-related cycle similar to that of the aggregate marriage rates. War-time tends to be associated with abnormally low birth rates and the post war with birth-rate booms. At first glance, marital birth hazards after WW1, shown by age group in Panel B of Figure 12, appear to follow this pattern: in 1920 they peaked at roughly 65% above their

[16] *La loi d'encouragement aux familles nombreuses* was passed in 1923; incentives included maintenance payments to large families, baby bonuses, reductions in the price of bread and train fares for families of three or more children, and social assistance to defray the cost of child care. In the 1920s, the *Médaille de la famille Française* was awarded to mothers of large families. As described in Roberts (1994), several of these measures pre-date the passage of the bill.

Figure 12: Births



A – Aggregate Birth Rate

B – Birth Hazard of Married Women

Source: INSEE, Mitchell (1998) and authors' calculations.

pre-war trends. However the birth hazards per married woman differ from the per-capita birth rates in an unexpected and revealing way: rather than returning to trend, the birth hazards remained 25% above their trends well into the 1930s. This is a large and persistent shift in marital birth-hazard rates that, particularly in the later years, extends to women too young to be directly affected by the war-time births deficit.

Higher post-war fertility incentives would in our model generate an increase in marriage surplus and thus generate higher post-war marriage rates. Our baseline results, described above, showed a much smaller birth-rate shift after the war; could the magnitude of the rise in married birth hazard rates account for the portion of the post-war marriage hazard peaks that remain unexplained in our model? To answer this, we recompute the calibrated model with an additional shock: a post-war shock to fertility incentives. Instead of letting the fertility preference parameters, σ_F (a_F), return to steady state values after the war, we calibrate the two post-war values so that the model matches the increase in the birth hazard. At the same time, we recalibrate the war-time shock, so that, as in the baseline experiment, the model matches the war-time marriage-rate decline.

The results, shown in Table 5, confirm the impact of the post-war fertility shock, not just for

Table 5: Results, Post-War Natality Shock

	Marriage hazard					
	Men		Women		Birth hazard	
	20-29	30-39	20-29	30-39	20-29	30-39
War time decline (model/data)	1.11	1.03	1.02	1.02	0.99	0.96
1919 peak (model/data)	0.57	0.77	1.01	1.04	0.80	0.17
1920 peak (model/data)	0.31	0.46	0.32	0.39		
Months to 50% of peak (model/data)	0.44	0.79	0.47	0.42		

Note: Analysis based on model recalibrated with post-war natality shock.

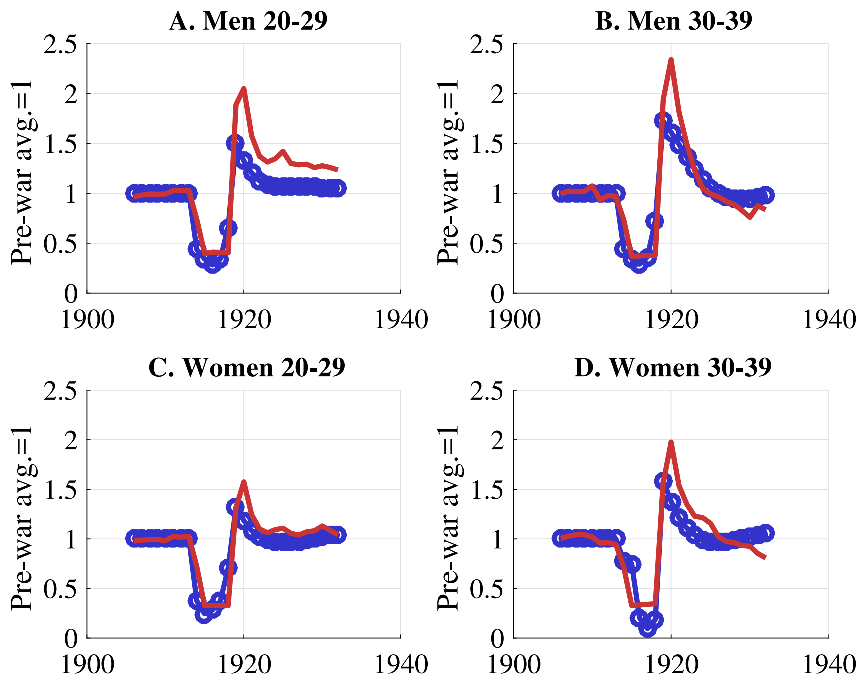
the birth hazards, but also for marriage.The post-war increase in birth hazards for young women rises, from the baseline 12%, to 80% of the actual peak. The effect on female marriage hazards is equally dramatic: the model now accounts for roughly 100% (versus the baseline of 66% for ages 20-29 and 74% for ages 30-39) of the post-war peaks in marriage hazard for women. Figure 13 shows the transition paths for marriage hazards in this experiment. The model also accounts for larger shares of the male marriage hazards, 57% and 77% of the post-war peaks (versus 42% and 61% in the baseline)[17]. Thus, augmenting the baseline model with the post-war fertility shock, namely a 70% reduction in the cost of children, can account for *all* of the post-war peaks in marriage hazards for women, as well as 80% of the post-war birth-rate peak of young women.

8 Conclusion

This paper documented a recurring pattern in marriage rates; they increase sharply after wars, despite the reduction in the male-female ratio caused by military mortality. For the case of France after World War 1, we were able to show that this was due largely to an anomalous and persistent rise in the female marriage hazard, contrary to the implication of standard matching

[17] The asymmetry by sex in the explained share of the hazard rate appears to be due in part to a post-war rise in marriage rates among teenage females, which is not accounted for in our calibration.

Figure 13: Marriage Hazards During the Transition with Post-War Natality Shock



Note: Marriage hazards during the transition, relative to their pre-war average; data (−) and model (∘).
Source: INSEE, Mitchell (1998) and authors' calculations.

models that the fall in the sex ratio should reduce the female marriage hazard.

To resolve this tension between theory and data, we proposed that composition effects generated by the war-time marriage bust could explain the post-war rise in the female marriage hazard. We formalized this idea in a dynamic model of two-sided matching. In our model, stochastic aging plays a central role; heterogeneity in marriage rates arises from changes in preferences and fertility over the life cycle, but the link between preferences and age is indirect and endogenous; the composition effect that we study is due to the war distorting the usual age-composition relationship.

We calibrated the model so that the steady state marriage and birth age profiles resembled those of pre-war France. The marriage-rate data implies an important asymmetry by sex: aging in the model tends to increase the marriage rates of young men and reduce those of young women. To represent the impact of the war, we replicated in the calibrated model the war-time marriage bust along with increased male mortality. The result was, for any given age group, a large post-war abundance of men in a high-marriage-gains state. This caused the female marriage hazards to increase, despite a lower post-war sex ratio. When combined with the post-war rise of pro-natalism, our mechanism can explain all of the rise in female marriage hazards.

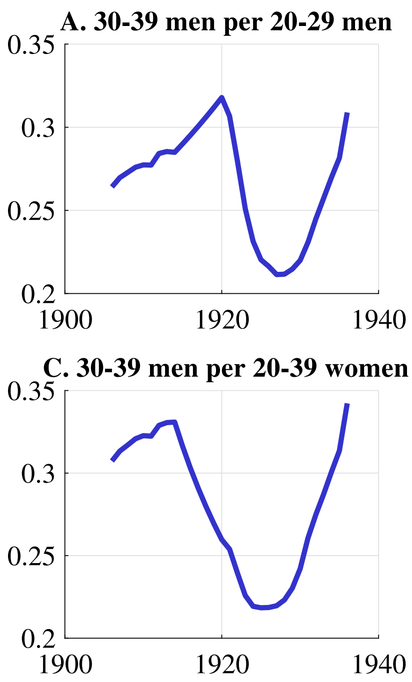
Our results do not conflict with previous work on the decline in the sex ratio. To the extent that local sex-ratio variation is independent of the composition effect, we would expect our model—augmented to include social-status heterogeneity—to match the results of Abramitzky et al. (2011): in areas with low-post-war sex ratios, women in our model would be less likely to marry, would marry lower-ranked men, and marry more slowly. However to the extent that the local sex ratio is negatively correlated with the length of marriage-market disruption, then our results would suggest an alternative interpretation of the finding that women marry down; the husbands are likely to have been better on dimensions unobservable in the data.

A more general point is that variation in the sex ratio is of singles is not necessarily a reliable indicator of marriage-market conditions; to the extent that such variation reflects the recent history of the matching process, the composition of the market can undermine or even overwhelm the direct effect of the sex ratio.

A The Aging Hypothesis

One effect of the war-time marriage bust was to shift the age distribution of single men to the right. We show this in Panel A of Figure 14 where the fraction of single men who are aged 30-39 is seen to increase from 0.28 to 0.32 postwar. Since older single men are more likely to marry, could this account for some of the post-war boom in the female marriage hazard? Panels B and C show why this cannot be: the number of single women postwar increased more than the population of older males, and so the ratio of single older men to single women was actually much lower in the post-war aftermath. The ratio of older single men to single women aged 20-29 fell from 0.35 to 0.32 and the ratio of of older men to single women aged 30-39 fell from 0.30 to 0.25. The positive effect of the rise in age of single males would be swamped therefore by the negative effect of the increase in the number of single women.

Figure 14: Aging of the Single Male



Note: Fraction of single men in each age group, as reported in the Census.
Source: INSEE and authors' calculations.

B The Deferred Marriage Hypothesis

In this section we consider the "deferred-marriage" hypothesis. This is the notion that matches between men and women kept forming during the war as they did before, but that they were not registered as marriages right-away. An abundance of de-facto marriages and their sudden registration at the war's end could explain, according to this hypothesis, the post-war peaks in marriage hazard rates. We argue below that this hypothesis does not provide a plausible account of the post-war peak in marriage hazards. We present three different arguments:

1. The matching between spouses was different after the war. Henry (1966) shows that women after the war were more likely than before to marry younger men as well as widowed/divorced men. Similarly, Abramitzky et al. (2011) show that women after the war were more likely to marry lower-status men particularly in areas that had been more strongly affected by the war. If matches formed during the war as they did in peace time, and only their registration got delayed, it is not clear why the type of matches should have changed.

2. The marriage deficit persisted throughout the entire 1920s. Panel A of Figure 15 shows the cumulative deviations of the number of marriages from its pre-war mean. A deficit of 900,000 marriages had accumulated by the end of the war, and it took until 1930 for these missing marriages to be replaced. If the missing marriages were all de-facto marriages that only needed registration, then why should replacement need 10 years?

3. The number of out-of-wedlock births also sheds light on the issue. The idea exploits the fact that de-facto marriages are generally accompanied by out-of-wedlock births. Consider out-of-wedlock fertility, that is the number of out-of-wedlock births per single women aged 15-45. Denote it by o_t/s_t, where o_t is out-of-wedlock births and s_t is the number of single women. Out-of-wedlock fertility can be decomposed as: $\ln(o_t/s_t) = \ln(o_t/d_t) + \ln(d_t/s_t)$, where d_t is the number of single women in de-facto marriages. Thus, variations in o_t/s_t

may result from variations in the birth rate of women in de-facto marriages, o_t/d_t, and from variations in the proportion of single women in de-facto marriages, d_t/s_t. We do not observe the birth rate of women in de-facto marriages, but assuming that it is proportional to the birth rate of married women, we can evaluate its contribution to changes in out-of-wedlock fertility. To this end, we estimate the following model:

$$\ln\left(\frac{o_t}{s_t}\right) = \alpha_0 + \alpha_1 \times t + \alpha_2 \times \ln\left(\frac{\ell_t}{m_t}\right) + \epsilon_t,$$

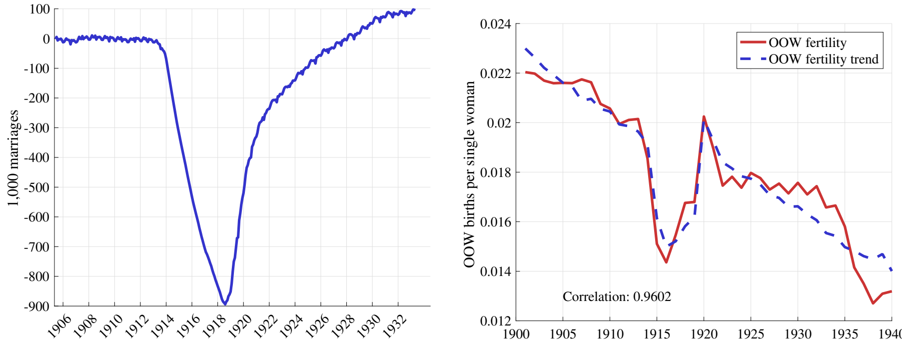
where ℓ_t denote legitimate births and m_t denote the number of married women. Panel B of Figure 15 shows that almost all (96%) the variation in out-of-wedlock fertility is explained by this model. The remaining variation can be attributed to changes in the proportion of single women in de-facto marriages, and possibly other missing factors.

How many out-of-wedlock births can be attributed to changes in the proportion of single women in de-facto marriages? Between 1914 and 1919 there was a total of 362,948 out-of-wedlock births. The model implies a total of 361,444. The difference between these figures results from changes in the proportion of single women in de-facto marriages (and possibly other missing factors) and is not significantly different from 0. We conclude that there was no significant changes in the proportion of single women in de-facto marriages during the war.

C Computation of Hazard Rates

In this section we derive cohort- and age-specific birth, marriage, divorce and death hazard rates. We show that, under some assumptions, a linear combinations of these flow rates determine the growth rate of the stocks of single, married, divorce and widow. Since these stocks are reported in the French Census for each year and age, from 1905 to 1935 (excluding 1914-1919), we can solve this linear system to compute the flow rates. For 1914-1919, which includes both the

Figure 15: Cumulative deviations in the number of marriages and out-of-wedlock fertility



A – Cumulative deviations in marriages

B – Out-of-wedlock fertility

Note: The model in Panel B is $o_t/s_t = \exp(\hat{\alpha}_0 + \hat{\alpha}_1 \times t + \hat{\alpha}_2 \times \ln(\ell_t/m_t))$ where $\hat{\alpha}_0$, $\hat{\alpha}_0$ and $\hat{\alpha}_2$ are estimated via Ordinary Least Squares. (See text for a definition of the symbols).
Source: Bunle (1954, p. 248), INSEE and authors' calculations.

war and one post-war year, we use additional assumption that we describe later. Our basic assumptions are:

1. Hazard rates do not depend on an individual's history.

2. Death probability of spouses are independent of each other.

3. The probability of any two events happening at the same time to the same person is zero. A person might, for instance, divorce or die in a given year, but not both.

4. The ages of spouses are equal. The probability that a married woman becomes a widow is equal to the death probability of a man her age.

Since the calculations below are identical for any birth cohort b at age a (therefore in calendar year $b + a$), we omit cohort and age indexes in our notation. We use the notation $N_s(m)$ to denote the stock of population (of a particular birth cohort at a particular age) that is of sex $s \in \{H, F\}$ and is in marital status $m \in \{S, M, D, W\}$ at the beginning of the calendar year t. The marital status can be single (S), married (M), divorced (D) and widow (W). Single (S) is restricted to never-married. Since our notation is specific to a particular cohort, it is understood that whenever $N_s(m)$ refers to birth cohort b at age a in date t, the notation $N_s'(m)$ refers to the same cohort at age $a+1$ in date $t+1$. We denote the marriage hazard by m_s, the death hazard by d_s, the immigration hazard by i_s and the divorce hazard by x_s. We use $\sim s$ to denote the sex opposite to sex s. There are 4 unknown hazard rates to compute each year for each age and each sex.

C.1 Peace-time Computations

For convenience, define $k_s \equiv 1 - d_s + i_s$. A widow in the next period may be a widow in the current period and survive but not marry; immigrate as a widow and not marry; or be married in the current period and have a spouse who dies. This translates into the law of motion for the stock of widows of sex s:

$$N_s'(W) = N_s(W)(k_s - m_s) + N_s(M)\, d_{\sim s}. \tag{23}$$

A person who will be married in the next period may already be married in the current period and survive along with his/her spouse, but must not have divorced in the current period. Alternatively, a married person in the next period may have been single, widowed or divorced in the current period and marry. The couple may also have immigrated to France already married. This translates into the law of motion for the stock of married people of sex s:

$$N_s'(M) = N_s(M)(k_s - d_{\sim s} - x_s) + m_s(N_s(S) + N_s(D) + N_s(W)). \tag{24}$$

A person who will be single (never-married) in the next period must be single in the current period, not die and not marry. Alternatively the person could have immigrated as a single. This translates into the law of motion for the stock of single people of sex s

$$N_s'(S) = N_s(S)(k_s - m_s). \tag{25}$$

Finally, a divorced person in the next period must be divorced (or immigrate already divorced) in the current period, not die and not marry. Alternatively, a divorced person in the next period could be married in the current period and divorce. This translates into the law of motion for the stock of divorced people of sex s:

$$N_s'(D) = N_s(D)(k_s - m_s) + N_s(M)\,x_s. \tag{26}$$

These equations can be written as

$$\begin{aligned}
\frac{N_s'(W)}{N_s(W)} - 1 &= -d_s + i_s - m_s + \frac{N_s(M)}{N_s(W)} d_{\sim s}, \\
\frac{N_s'(M)}{N_s(M)} - 1 &= -d_s + i_s - d_{\sim s} - x_s + \frac{N_s(S) + N_s(D) + N_s(W)}{N_s(M)} m_s, \\
\frac{N_s'(S)}{N_s(S)} - 1 &= -d_s + i_s - m_s, \\
\frac{N_s'(D)}{N_s(D)} - 1 &= -d_s + i_s - m_s + \frac{N_s(M)}{N_s(D)} x_s.
\end{aligned}$$

Note that the first equation for sex s involve the death rate $d_{\sim s}$. In order to compute the 8 unknown rates, the systems for each sex must be solved simultaneously. The hazard rates for a given cohort and age are given by:

$$\begin{bmatrix} d_H \\ i_H \\ m_H \\ x_H \\ d_F \\ i_F \\ m_F \\ x_F \end{bmatrix} = X^{-1} \begin{bmatrix} N_H'(W)/N_H(W) - 1 \\ N_H'(M)/N_H(M) - 1 \\ N_H'(S)/N_H(S) - 1 \\ N_H'(D)/N_H(D) - 1 \\ N_F'(W)/N_F(W) - 1 \\ N_F'(M)/N_F(M) - 1 \\ N_F'(S)/N_F(S) - 1 \\ N_F'(D)/N_F(D) - 1 \end{bmatrix},$$

where the X matrix is defined below. There is a special case when an age-sex group has no married people: $N_s(M) = 0$. This can be the case for young people. Then the divorce rate x_s is undefined (say 0). In this case we have

$$N_s'(S) = N_s(S)(1 - d_s + i_s - m_s)$$
$$N_s'(M) = m_s(N_s(S) + N_s(D) + N_s(W))$$

The last equation can be used to compute the marriage rate. The death and immigration rates, however, cannot be separately identified. In practice this case occurs only with the youngest ages 14 and 15, so it is reasonable to set the death rate to zero for these age groups, given their low natural mortality. Then the immigration rate follows from the first equation.

$$X = \begin{bmatrix} -1 & 1 & -1 & 0 & \frac{N_H(M)}{N_H(W)} & 0 & 0 & 0 \\ -1 & 1 & \frac{N_H(S)+N_H(D)+N_H(W)}{N_H(M)} & -1 & -1 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & \frac{N_H(M)}{N_H(D)} & 0 & 0 & 0 & 0 \\ \frac{N_F(M)}{N_F(W)} & 0 & 0 & 0 & -1 & 1 & -1 & 0 \\ -1 & 0 & 0 & 0 & -1 & 1 & \frac{N_F(S)+N_F(D)+N_F(W)}{N_F(M)} & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & \frac{N_F(M)}{N_F(D)} \end{bmatrix}.$$

C.2 War-time Computations

The period during which the population stocks by marital status are not observed extends from January 1915 to January of 1920. Thus, we cannot use the method described above to compute hazard rates for these years. We then proceed as follows. First, we use cohort-specific military death rates reported by Vallin (1973) for mortality during the war. Second, we assume that age-specific marriage hazards during the war are multiplied by a set of shifters that remain constant for the duration of the war. Third, we are missing one-post-war year, 1919, so we allow the marriage shifters in 1919 to take on a different set of values. Fourth, the age-specific divorce and immigration rates are assumed to remain at their average for 1910-1913, as derived from the peace-time procedure described above.

There is a total of 8 shifters to determine. One marriage shifter common to all men between the age of 20 and 29 during the war, and one for 1919; one marriage shifter common to all men 30-39 during the war, and one for 1919; and similarly for women.

Under these assumptions, and for a guess of the marriage shifters during the war and in 1919, we can compute the population stocks by age, sex and marital status in 1920 by starting from the population at the start of 1914. The predicted 1920 population requires one modification: the population of France in the post war was increased 4% by the addition of Alsace-Lorraine. We increase the 1919 population by 4% in our calculation (implicitly assuming that the composition of the Alsace-Lorraine population was the same as that of the rest of France).

We choose the 8 shifters to minimize a score function that consists of the distance between the predicted 1920 population by age and marital status and its empirical counterpart, plus the distance between the observed per-capita marriage rate (Mitchell, 1998) and that predicted by the shifters.

D Equilibrium: Definition and Computation

A dynamic competitive-search equilibrium of our model is a sequence of:

1. Value functions

   (a) for men, $\{R_{H,t}(a_H), W_{H,t}(a_H), V_{H,t}(a_H), v_{H,t}(a_H)\}$,

   (b) for women, $\{R_{F,t}(a_F), V_{F,t}(a_F), v_{F,t}(a_F)\}$,

2. utility offers, $\{w_t(a_F, a_H)\}$, participation thresholds, $\{\xi_t^*(a_H)\}$,

3. queue lengths, $\{\phi_t(a_F, a_H)\}$,

4. populations of single men, $\{P_{H,t}(a_H)\}$, and single women, $\{P_{F,t}(a_F)\}$,

such that at each date t:

1. Men and women are optimizing:

   (a) Women solve problem (16)-(17).

   (b) The marginal man is indifferent between participating or not: $\xi_t^*(a_H) \leq v_{H,t}(a_H)$.

   (c) participating men are indifferent between sub-markets, Equation (15).

2. The assignment of agents to sub-markets is feasible, Equation (20).

3. The population of singles follows the laws of motion in Equations (11)-(14).

D.1 Computations

1. Choose

   (a) Initial conditions $P_{H,1}(a_H)$ and $P_{F,1}(a_F)$.

   (b) Terminal conditions $W_{H,T}(a_H)$ and $V_{F,T}(a_F)$.

2. Guess a path $\{\phi_t(a_F, a_H), v_{H,t}(a_H)\}$

3. At date $t = 1, 2, \ldots T-1$

   (a) Use $\phi_t(a_F, a_H)$, $P_{H,t}(a_H)$, and $P_{F,t}(a_F)$ to compute marriage rates $\pi_{H,t}(a_H)$ and $\pi_{F,t}(a_F)$. Use Equations (2)-(5).

   (b) Use $\pi_{H,t}(a_H)$, $\pi_{F,t}(a_F)$, and $v_{H,t}(a_H)$ to compute $P_{H,t+1}(a_H)$ and $P_{F,t+1}(a_F)$. Use Equations (11)-(14).

4. At date $t = T-1, T-2, \cdots, 1$

(a) Compute $R_{H,t}(a_H)$ and $R_{F,t}(a_F)$. Use Equations (10) and (7).

(b) Compute $x_t(a_F, a_H)$. Use Equation (18).

(c) Solve the static problem given $\{x_t(a_F, a_H), P_{F,t}(a_F), P_{H,t}(a_H)\}$. This yields updated values $\phi_t^{new}(a_F, a_H)$ and $v_{H,t}^{new}(a_F)$.

(d) Compute $W_{H,t}(a_H)$. Use Equation (9).

5. Compare $\left\{\phi_t^{new}(a_F, a_H), v_{H,t}^{new}(a_H)\right\}$ to the initial guess. Update if the difference is larger than a tolerance threshold, and go to step 3.

E Calendar-Age Statistics from the Model

Consider men in steady state. There are four possible states a man can be in: single in stage 1, 2 and 3 and married (the 4th state). Let

$$\lambda_H^\tau = \left[\lambda_{H,1}^\tau, \lambda_{H,2}^\tau, \lambda_{H,3}^\tau, \lambda_{H,4}^\tau\right]$$

represent the mass of men in each one of these states, at calendar age τ, in the steady state. By definition $\lambda_H^{18} = [\chi_H, 0, 0, 0]$: an initial condition. Then we have

$$\lambda_H^{\tau+1} = \lambda_H^\tau T_H$$

where the transition matrix T_H is

$$T_H = \begin{bmatrix} (1-\delta_H(1))(1-\pi_H(1)\mu(1)) & \delta_H(1)(1-\pi_H(1)\mu(1)) & 0 & \pi_H(1)\mu(1) \\ 0 & (1-\delta_H(2))(1-\pi_H(2)\mu(2)) & \delta_H(2)(1-\pi_H(2)\mu(2)) & \pi_H(2)\mu(2) \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

This computation allows us to build statistics that are calendar-age specific in steady state.

During the transition the matrix T_H varies every year, but the logic remains the same. The logic is also the same for women, but there are more possible states since the life-stage of woman determines her ability to have children. Thus, women may be single or married in stage 1, 2 or 3, and, when married, they may have 0, 1, 2 or 3 children.

F Calibrated Parameter Values

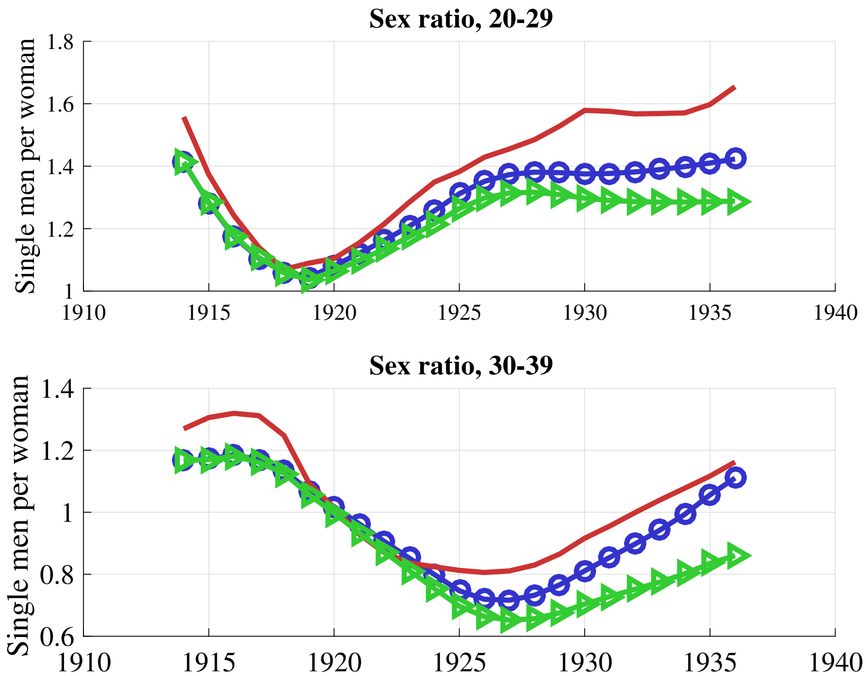
Table 6: Calibrated Parameters

Demography	
Discount factor	$\beta = 0.96$
Male transition	$\delta_H(a_H) = (0.14, 0.03)$
Flow of stage-1 men	$\chi_H = 1.00$
Female transition	$\delta_F(a_F) = (0.13, 0.01)$
Flow of stage-1 women	$\chi_F = 1.00$
Single	
Male output	$y_H^S(a_H) = (16.52, 0.79, 0.00)$
Female output	$y_F^S(a_F) = (-0.02, 1.45, 0.51)$
Participation cost	
Mean	$\mu_\xi = 4.18$
Std. dev.	$\sigma_\xi = 0.65$
Marriage	
Output param.	$\omega = 6.86$
Cost elasticity	$\gamma_A(a_F) = (2.79, 3.11)$
Cost fixed	$\gamma_K(k) = (118.47, 60.30, 80.48)$

G The initial sex ratio

In all of the above transition-path analysis, the sex ratio of entrants (i.e. 18-year olds) is exogenously set each year at the empirical value computed from the census data, while in the steady-state value it is set to 1.0, approximating the pre-war average. To see how this affects our main results, we run a new experiment in which we compute the transition of the model holding

Figure 16: Sex Ratios of Single



Note: Single men per woman. Data (−), baseline model (∘), model with constant initial sex ratio (▷).
Source: Authors' calculations.

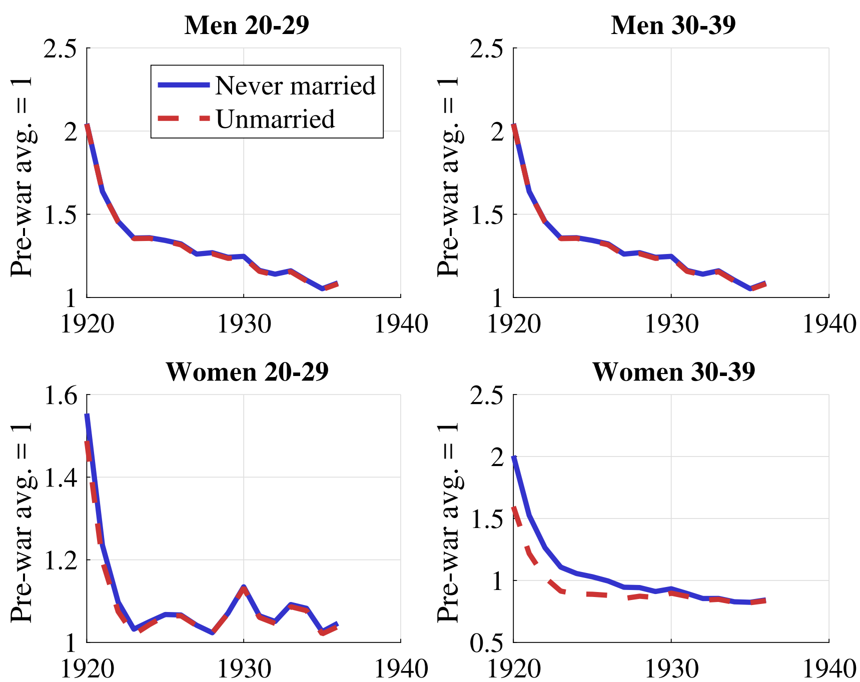
the sex ratio of entrants constant at its steady-state value. Figure 16 shows that allowing the sex ratio of entrants to change permits a better match of the observed sex ratios of singles for 20-29 and 30-39 age groups. It turns out that this also contributes to the ability of the model to explain the 1919 peak of marriage hazards for 20-29 year-old women; holding the sex ratio constant reduces the explained share of the peak from 66% to 48%, and for the older group from 74% to 48%. There is virtually no corresponding impact on the male hazards. Since it is likely that similar forces are operation on the under-18s as on the older women, this tell us that incorporating the under-18 population into the matching market would likely increase the fraction of the female marriage-hazard explained by composition effects[18].

[18]On their own, however, the initial sex ratios do not account for higher female hazards: when the initial sex ratio of entrants is the only changing variable in the transition, we find that the model generates little to no changes in marriage hazards. The results of this experiment are available upon request.

H Widows

The war increased the fraction of widows, particularly in the age 30-39 group. If widows have a higher re-marriage rate, this could partially explain the post-war peak in female marriage hazards. The data allows us to distinguish unmarried (including widows and divorced spouses) from never-married individuals. So, in Figure 17 we show that marriage hazards are very similar for previously married singles. Hence, neither the existence nor the magnitude of the post-war peaks in marriage hazards can be explained by an increase in the previously-married share of singles.

Figure 17: Postwar Marriage Hazards for Never-married and Unmarried



Note: The hazards are computed by dividing the flow of marriages for an age-sex group in a given period, as reported by Bunle (1954), by the stock of never-married, or unmarried at the begining of the period, as reported in the Census. The figures are normalized by their pre-war averages.
Source: Bunle (1954), INSEE and authors' calculations.

References

ABRAMITZKY, R., A. DELAVANDE AND L. VASCONCELOS, "Marrying Up: The Role of Sex Ratio in Assortative Matching," *American Economic Journal - Applied Economics* 3 (2011), 124–157.

ANGRIST, J., "How Do Sex Ratios Affect Marriage and Labor Markets? Evidence from America's Second Generation," *The Quarterly Journal of Economics* 117 (2002), 997–1038.

BECKER, G. S., "A Theory of Marriage: Part I," *Journal of Political Economy* 81 (1973), 813–846.

BETHMANN, D. AND M. KVASNICKA, "World War II, Missing Men and Out of Wedlock Childbearing," *The Economic Journal* 123 (2014), 162–194.

BOTTICINI, M. AND A. SIOW, "Are there Increasing Returns in Marriage Markets?" Unpublished, University of Toronto, Department of Economics, 2008.

BRAINERD, E., "The Lasting Effect of Sex Ratio Imbalance on Marriage and Family: Evidence from World War II in Russia," *Review of Economics and Statistics* 99 (2017), 229–242.

BRONSON, M. A. AND M. MAZZOCCO, "Cohort Size and the Marriage Market: Explaining a Century of Changes in U.S. Marriage Rates," *UCLA CCPR Working Papers* 2018.

BUNLE, H., *Le mouvement naturel de la population dans le monde de 1906 à 1936* (Paris: Les éditions de l'INED, 1954).

BURDETT, K. AND M. G. COLES, "Long-Term Partnership Formation: Marriage and Employment," *Economic Journal* 109 (June 1999), F307–34.

CHOO, E., "Dynamic Marriage Matching: An Empirical Framework," *Econometrica* 83 (2015), 1373–1423.

Choo, E. and A. Siow, “Who Marries Whom and Why,” *Journal of Political Economy* 114 (February 2006), 175–201.

Groves, E. R. and W. F. Ogburn, *Marriage and Family Relations* (New York, Henry Holt & Co, 1928).

Henry, L., “Perturbations de la nuptialité résultant de la guerre 1914-1918,” *Population* 21 (1966), 273–332.

Huber, M., *La population de la France pendant la guerre* (Paris: Les Presses Universitaires de France, 1931).

Julien, B., J. Kennes and I. King, “Bidding for Labor,” *Review of Economic Dynamics* 3 (2000), 619–649.

Kennes, J. and J. A. Knowles, “Liberalization of Birth Control and the Unmarried Share of Births: Evidence from Single Mothers in the Marriage Market,” Unpublished, Simon Fraser University, Department of Economics, 2015.

Menzio, G. and S. Shi, “Directed Search on the Job, Heterogeneity, and Aggregate Fluctuations,” *American Economic Review* 100 (May 2010), 327–32.

Mitchell, B. R., *International Historical Statistics: Europe, 1750-1993* (New York, NY: Stockton Press, 1998).

Moen, E., “Competitive Search Equilibrium,” *Journal of Political Economy* 103 (1997), 385–411.

Pissarides, C. A. and B. Petrongolo, “Looking into the Black Box: A Survey of the Matching Function,” *Journal of Economic Literature* 39 (June 2001), 390–431.

Regalia, F. and J.-V. Ríos-Rull, "What Accounts for the Increase in Single Households and the Stability in Fertility?" Unpublished, Universitat Pompeu Fabra, Department of Economics, 1999.

Robert, J.-L., "Women and Work in France During the First World War," in R. Wall and J. Winter, eds., *The Upheaval of War: Family, Work and Welfare in Europe, 1914-1918,* chapter 9 (Cambridge: Cambridge University Press, 2005), 251–266.

Roberts, M. L., *Civilization without Sexes: Reconstructing Gender in Postwar France 1917-1927* (University of Chicago Press, 1994).

Shi, S., "A Directed Search Model of Inequality with Heterogeneous Skills and Skill-Biased Technology," *The Review of Economic Studies* 69 (2002), 467–491.

Shimer, R., "The Assignment of Workers to Jobs in an Economy with Coordination Frictions," *Journal of Political Economy* 113 (2005), 996–1025.

Shimer, R. and L. Smith, "Assortative Matching and Search," *Econometrica* 68 (2000), 343–369.

Vallin, J., *La mortalite par generation en France, depuis 1899* (Paris: Les éditions de l'INED, 1973).

Vandenbroucke, G., "Fertility and Wars: The Case of World War I in France," *American Economic Journal, Macroeconomics* 6 (April 2014), 108–136.

Vincent, P., "Conséquences de six années de guerre sur la population française," *Population* 1 (1946), 429–440.

Wright, R., P. Kircher, B. Julien and V. Guerrieri, "Directed Search: A Guided Tour," NBER Working Paper No. 23884, 2017.